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5702

M.A./M.Sc. (Sem. I) Examination, Dec. 2022

MATHEMATICS

Paper – Math 1C2

(Complex Analysis or Analysis-I)

Time Allowed : Three Hours

Max. Marks : 100

This question paper will be divided into ***three*** *sections as under :*

Section-A *Max. Marks-10*

This section contains one compulsory question ***with 10*** *parts, having* ***2*** *parts from each unit, short answer in* ***20*** *words for each part. All questions carry equal marks.*

Section-B ***Max. Marks-50***

This section contains ***10*** *questions having* ***2*** *questions from each unit. Answer* ***5*** *questions (****250*** *words each) selecting* ***one*** *question from each unit. All questions carry equal marks.*

Max. Marks-40

Section-C

This section contains 4 descriptive type questions (questions may have sub-division) covering all units but not more than one question from each unit. Answer any two questions. (500 words each). All questions carry equal marks.

Section A

1 (i) **Define Analytic function.**

(ii) **Define Conformal Mapping.**

(iii) **Define complex integration.**

(iv) **Define Taylor's series.**

(v) **Write statement of Morera's theorem.**

(vi) **Write statement of Maximum Modulus Principle theorem**

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(vii) Define Singularities.

(viii) Define Branch Points.

(ix) Write statement of Rouche's theorem.

(x) Define meromorphic function.

Section B

UNIT-I

2. Prove that the function $u = e^x (x \cos y - y \sin y)$ satisfies Laplace's equation and find the corresponding analytic function $f(z) = u + iv$.

3. Prove that in the general in the bilinear transformation :

$$w = \frac{az + b}{cz + d}$$

there are two values of z for which $w = z$, but there is only one if $(a - d)^2 + 4bc = 0$, show that, if there are distinct invariant points p and q, the transformation may be put in the form :

$$\frac{w - p}{w - q} = k \frac{z - p}{z - q};$$

and that if there is only one invariant point p, the transformation may be put in the form

$$\frac{1}{w - p} = \frac{1}{z - p} + k.$$

UNIT-II

4. Prove that if a function $f(z)$ is an analytic function within and on a closed contour c and a is any point lying it it, then : https://www.uokonline.com

$$f'(a) = \frac{1}{2\pi i} \int_c \frac{f(z)}{(z - a)^2} dz$$

5. Expand $\frac{1}{z(z^2 - 3z + 2)}$ in Laurent's series for the regions :

(a) $0 < |z| < 1$

(b) $1 < |z| < 2$

(c) $|z| > 2$.

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UNIT-III

6. Write statement and proof of Liouville's theorem.

7. Write statement and proof of Schwartz's-lemma.

UNIT-IV

8. Write statement and proof of Riemann's theorem.

9. Determine the nature of the pole at the origin of the function :

$$\frac{e^z}{z \sin mz}$$

UNIT-V

10. Prove that if $f(z)$ and $g(z)$ are analytic within and on a closed curve c, and $|g(z)| < |f(z)|$ on c, then $f(z)$ and $f(z) + g(z)$ have same number of zeros inside c.

11. Find the residues of $\frac{1}{\sinh z}$ at $z = in\pi$.

Section C

12. (a) Behaviour of power series $\sum \frac{z^{4n}}{4n+1}$ on the circle of convergence.

(Contd....

(b) State and prove Cauchy's inequality.

13. (a) State and prove Liouville's theorem.

(b) Prove that a function which has no singularity in the finite part of the plane or at infinity is constant function.

14. (a) Use method of contour integration to prove that :

$$\int_0^{2\pi} \frac{d\theta}{1 + a^2 - 2a\cos\theta} = \frac{2\pi}{1 - a^2}, \quad 0 < a < 1$$

(b) State and prove Taylor's theorem.

15. (a) Evaluate by Cauchy's integral formula

$$\int_c \frac{dz}{z(z + \pi i)}, \text{ where } c \text{ is } |z + 3i| = 1.$$

(b) Prove that if $f(z)$ is a function of z analytic at all the points on and inside a closed contour c, then :

$$\int_c f(z)dz = 0.$$

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