QC-5701

M. A./M. Sc. (Sem.-I) Examination, December - 2023

MATHEMATICS

Paper - MATH - 1C1

Algebra – I

Time Allowed : **Three Hours**

Maximum Marks : **100**

Note : This question paper contains **two** sections as under :

https://pyqlelo.com

Section-A ***Max. Marks-20***

*This section will have **10** questions, having **02** questions from each unit with no internal choice. Answer in about **20** words. All questions will have equal marks.* ***(10×02=20)***

Section-B ***Max. Marks-80***

*This section will have **10** questions, having **02** questions from each unit, one question to be attempted from each unit. Words limit approximately **400-500** words. Questions may have sub divisions. All questions will have equal marks.* ***(16×05=80)***

https://pyqlelo.com

SECTION – A

1 (i) Determine whether the mapping $\phi : R^2 \to R$ linear transformation.

(ii) State Cayley – Hamilton theorem.

(iii) Define Canonical forms.

(iv) Define Hermitian form.

(v) Define orthonormal sets.

(vi) State Sylow's theorem.

(vii) Define Normal series.

(viii) Define Ideals.

(ix) Define Conjugate elements.

(x) Define finite field extension.

https://pyqfield.com

SECTION - B

UNIT - I

2 Let $t : V \to V'$ be a linear transformation, then t is monomorphism if and only if $\ker(t) = \{0\}$.

OR

Show that the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$ satisfies. the Cayley - Hamilton theorem.

UNIT - II

3 A $n \times n$ matrix A over a field F is invertable if and only if rank $(A) = n$

OR

Prove that distinct non-zero characteristics vectors of a linear operator t corresponding to distinct characteristic value of t are linearly independent.

UNIT - III

4 Prove that for $u, v \in R^2$ where

$u = (a_1, a_2), v = (b_1, b_2)$ and

$< u, v > = b_1 (a_1 + 2a_2) + b_2 (2a_1 + 8a_2)$

defines and inner product.

OR

List all conjugate classes in the symetric group S_3 and verify the class equation.

UNIT - IV

5 Prove that any two composition series of a group G are equivalent.

OR

Prove that every Euclidean ring R is a principal ideal domain.

UNIT - V

6 Show that if $a \in K$ be a root of $p(x) \in F(x)$ where $F \subseteq K$. Then in $K\{x\}, (x-a)$ is a divisor of $p(x)$ i.e $(x-a)/p(x)$

OR

If K/F is finite extension, then it is an algebraic extension.

https://pyqlelo.com