U-5703

M.A./M.Sc. (Sem.-I) Examination, 2021

MATHEMATICS

PAPER-MATH 1 C(III)

MECHANICS

Time allowed : **Three Hours**

Maximum Marks : **70**
Min. Pass Marks : **28**

Note : This question paper will be divided into three sections as under :

https://www.uokonline.com

Section-A **Max. Marks-10**

Will carry **10** *marks with* **01** **compulsory** *question comprising* **10** *short answer type questions (maximum 20 words answer) taking two questions from each unit. Each question shall be of one mark.*

Section-B **Max. Marks-25**

Will carry **10** *questions.* **02** *questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question from each unit.*

Section-C **Max. Marks-35**

Will carry 35 marks with five long answer type qustions comprising one compulsory question (question no. 07) of **15** *marks and four questions of 10 marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in 500 words) and any two more questions (answer about in* **100** *words) out of remaining four questions.*

https://www.uokonline.com

SECTION - A

1 (i) Define radius of gyration.

(ii) Define momental ellipsoide for a rigid body at any point O.

(iii) Write expression for moment of effective forces about the axis of rotation of a rigid body rotating about a fixed axis.

(iv) Define compound pendulum.

(v) Define generalized coordinates.

(vi) Define holonomic system.

(vii) Define eulerian angles.

(viii) Explain instantaneous axis of rotation

(ix) Define Hamilton's variables.

(x) Define Poisson's brackets.

SECTION - B

UNIT - I

2 If A and B be the M.I. of a uniform lemina about two perpendicular axes, OX and OY, lying in its plane and F be the product of inertia of the lemina about these lines, show that the principal moments at 0 are equal to

$$\frac{1}{2}\left[(A+B) \pm \sqrt{(A-B)^2 + 4F^2}\right]$$

OR

A rough uniform board, of mass m and length 2a, rests on a smooth horizontal plane and a man of mass M walks on it from one end to other, find the distance through, which the board moves in this time.

https://pyqlelo.com

UNIT - II

3 A uniform rod AB is freely movable on a rough inclined plane, whose inclination to the horizon is α and whose coefficient of friction is μ, about a smooth pin fixed through the end A, the bar held in the horizontal position in the plane and allowed to fall from the position, if θ be the angle through which it falls from rest show that

$$\frac{\sin\theta}{\theta} = \mu\cot\alpha$$

OR

A uniform semi circular arc, of mass m and radius a, is fired at its ends to two points in the same vertical line, and is rotating with constant angular velocity w, show that the horizontal thrust on the upper end is :

$$m\left[\frac{g+w^2a}{\pi}\right]$$

UNIT - III

4 The Lagrangian function has the form $L = \dot{q}_k q_k - \sqrt{1-\dot{q}_k^2}$ then show that the generalized acceleration is zero.

OR

Explain small oscillations by the use of Lagrange's equations.

UNIT - IV

5 Derive Hamilton's Canonical equations.

OR

A body moves under no forces, so that the resolved port of its angular velocity about one of the principal axis at the centre of gravity is constant. Show that the angular velocity of the body must be constant.

https://www.uokonline.com

UNIT - V

6 Deduce Lagrange's equations by using Hamilton's principle.

OR

Define Poisson's bracket and show that

(i) $[u + v, w] = [u, w] + [v, w]$

(ii) $[u, vw] = [u, v]w + v[u, w]$

SECTION - C

7 (a) Show that M.I. of a right solid cone whose height is h and radius of its base is a, is $\frac{3M}{20}\left[a^2 + 4h^2\right]$ about a line passing through the vertex parallel to its base.

(b) The lengths AB and AD of a of the sides of a rectangle ABCD are 2a and 2b, show that the inclination to AB of one of its principal axes at A is $\frac{1}{2}\tan^{-1}\left[\frac{3ab}{2\left(a^2 - b^2\right)}\right]$.

8 (a) Derive the expression for the moment of the effective forces about a fixed axis of rotation.

(b) A bent lever, whose arms are of length a and b, the angle between them being α, make small oscillations in its own plane about the fulcrum, prove that, the length of the corresponding simple pendulum is

$$\frac{2}{3}\left[\frac{a^3 + b^3}{\sqrt{a^4 - 2a^2 b^2 \cos\alpha + b^4}}\right]$$

9 Derive Lagrange's equations of motion in generalized coordinates for finite forces under the holonomic dynamical system.

10 Deduce Euler's dynamical equation of a rigid body moving about a fixed point.

11 State and prove principle of least action and also write the distinction between Hamilton's principle and principle of least action.