PB **5704**

Total No. of Pages: 04

Roll No. _____

M.A. / M.Sc. (Semester-I) Examination - Dec.-2025
Mathematics
Paper - MATH 1C4
Methods of Applied Mathematics-I

Time Allowed: Three Hours **Maximum Marks: 100**

This question paper contains two sections as under :

Section-A **Max. Marks-20**

This section will have 10 questions, having 02 questions from each unit with no internal choice. Answer in about 20 words. All questions will have equal marks. (2×10=20)

Section-B **Max. Marks-80**

This section will have 10 questions, having 02 questions from each unit, one question to be attempted from each unit. Words limit is approximately 400-500 words. Questions may have sub divisions. All questions will have equal marks. (16×05=80)

Section-A

1. i) Define linear integral equaition.

   ii) Define separable Kernel.

   iii) Define Resolvent kernel of Volterra integral equation of second kind.

   iv) Define iterated kernels of fredholm integral equation.

   v) Define laplace transfer m.

   vi) Find : $L^{-1}\left(\frac{4}{p-2}-\frac{3p}{p^2+16}+\frac{5}{p^2+4}\right)$

   vii) Define integral equation of convolution type.

   viii) Solve the following differential equation using laplace transform technique

   $(D^2+1)\,y = 0\,;\;\; y(0) = 1 = y^1(1)$

   ix) Define Generating Function for $J_n(x)$

   x) Define Legendre's polynomial.

Section-B

Unit-I

2. Define Volterra integral equaition of first kind and show that the $g(x) = xe^x$ is a solution of the Volterra integral equation.

$$g(x) = \sin x + 2 \int_0^x \cos(x-t)\, g(t)\, dt$$

OR

Define Fredholm integral equation of second kind and solve the fredholm integral equation of second kind given below.

$$g(x) = x + \lambda \int_0^1 \left(xt^2 + x^2 t\right) g(t)\, dt$$

Unit-II

3. Solve by method of successive approximation

$$g(x) = \frac{3}{2} e^x - \frac{1}{2} xe^x - \frac{1}{2} + \frac{1}{2} \int_0^1 t\, g(t)\, dt$$

OR

Using method of successive approximation, solve

$$g(x) = 1 + \int_0^x (x-t)\, g(t)\, dt \qquad \text{taking } g_0(x) = 0$$

Unit-III

4. a) Find Laplace transform of $x^2 e^x \sin 4x$

b) Evaluate : $L^{-1}\left\{\dfrac{p^2-6}{p^3+4p^2+3p}\right\}$

OR

a) Define Fourier transform and find Fourier sine transform of the following function

$$f(x) = \begin{cases} \sin x & ; \ 0 < x < a \\ 0 & , \ x > a \end{cases}$$

b) Find Fourier transform of the following function

$$f(x)=\begin{cases}1 & , |x|<a \\ 0 & |x|>a\end{cases}$$

and hence evaluate

i) $\int_{-\infty}^{\infty} \frac{\sin pa \cos px}{p} dp$

ii) $\int_{0}^{\infty} \frac{\sin p}{p} dp$

Unit-IV

5. Solve : $(D^3 - D^2 - D+1)\, y = 8te^t$

given that $y(0) = 0,\ y'(0) = 1,\ y''(0) = 0$

OR

solve the following integral equation

$$F(x)=\int_0^x \frac{g(t)}{(x-t)^a} dt \quad ; 0<a<1$$

Unit-V

6. a) If m is a positive integer, then show that

$${}_2F_1(-m, a+m; c; x)$$

$$=\frac{x^{1-c}(1-x)^{c-a}}{\Gamma(c+m)} \Gamma(c) \frac{d^m}{dx^m}\left[x^{c+m-1}(1-x)^{a+m-1}\right]$$

b) Prove that :

$$(c)_m \frac{d^m}{dx^m}\left[e^x {}_1F_1(a; c; x)\right]=(-1)^m (c-a)_m e^x {}_1F_1(a; c+m; x)$$

OR

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a) Show that :

$$2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$$

b) Show that :

$$(n+1)\, p_{n+1} = (2n+1)\, x\, p_n - n\, p_{n-1}$$

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