KU-5704

KU-5704

M.A./M.Sc. (Sem.-I) Examination, December - 2024

MATHEMATICS

Paper - MATH - 1C4

Methods of Applied Mathematics-I

Time Allowed : **Three Hours**

Maximum Marks : **100**

This question paper contains ***two*** *sections as under :*

https://pyqlelo.com

Section-A ***Max. Marks-20***

*This section will have **10** questions, having **02** questions from each unit with no internal choice. Answer in about **20** words. All questions will have equal marks. **(02×10=20)***

Section-B ***Max. Marks-80***

*This section will have **10** questions, having **02** questions from each unit, one question to be attempted from each unit. Words limit approximately **400-500** words. Questions may have sub divisions. All questions will have equal marks. **(16×05=80)***

https://myqeb.com

SECTION – A

1 (i) Define Homogenous Fredholm integral equation of second kind.

(ii) Define symmetric kernel with an example.

(iii) Find first two iterated kernels of the kernel

$$K(x,t) = (x-t)^2;\ a = -1,\ b = 1$$

(iv) Define Resolvent Kernel.

(v) State and prove first shifting property of Laplace transformation.

(vi) Write convolution theorem of Laplace transformation.

(vii) Using derivative formula, show that

$$L\{\cos ax; p\} = \frac{p}{p^2 + a^2}.$$

(viii) Evaluate $\int_0^\infty e^{-x} \sin x$ using Laplace transformation.

(ix) Write Rodrigues formula for Legendre's function.

(x) Show that $J_{+1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

SECTION - B

UNIT - I

2 Solve the homogenous Fredholm integral equation of the second kind

$$g(x) = \lambda \int_0^{2\pi} \sin(x+t) g(t) dt$$

OR

Solve the integral equation

$$g(x) = x + \lambda \int_0^1 (1 + x + t) g(t) dt .$$

UNIT - II

3 Solve $g(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 xt\, g(t) dt$ by the method of successive approximations.

OR

Using the method of successive approximations, solve the integral equation

$$g(x) = x - \int_0^x (x-t) g(t) dt , \; g_o(x) = 0 .$$

UNIT - III

4 (a) Find Laplace transform of the function $\sin\sqrt{t}$ and hence obtain Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$.

(b) Using convolution theorem find

$$L^{-1}\left\{\frac{p^2}{(p^2+a^2)^2}; t\right\}.$$

OR

(a) Find Fourier Casine transform of $\left(\frac{1}{1+x^2}\right)$.

(b) Find the Fourier transform of

$$f(x) = \begin{cases} x, & |x| \le a \\ 0, & |x| > a \end{cases}$$

UNIT - IV

5 Solve $(D^2+9)y=\cos 2x$; if $y(0)=1$, $y(\pi/2)=-1$ with the help of Laplace transform.

OR

Show that $\int_0^\infty \cos x^2 dx = \frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}$ using Laplace transformation.

UNIT - V

6 (a) If m is a positive integer, and $|x|>1$, show that ${}_2F_1\left(\frac{m+1}{2};\frac{m+2}{2};1;-\frac{1}{x^2}\right)$

$$=\frac{(-1)^m x^{m+1}}{m!}\frac{d^m}{dx^m}\left(\frac{1}{\sqrt{x^2+1}}\right)$$

(b) ${}_nP_n(x)=xP_n'(x)-p'_{n-1}(x)$.

OR

(a) Prove that

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}.$$

(b) $2nJ_n(x) = x\left[J_{n-1}(x) + J_{n+1}(x)\right]$.

---

https://pyqlelo.com