QM-5704

M.A./M.Sc. (Sem.- I) Examination, – December-2023

MATHEMATICS

PAPER – MATH-1C4

Methods of Applied Mathematics-I

Time allowed : **Three Hours**

Maximum Marks : **100**

Note : This question paper contains two sections as under :

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SECTION – A

1 (i) Define Singular integral equation with an example.

(ii) Define Separable kernel with an example.

(iii) Define orthogonal kernels with an example.

(iv) Define Neumann series for solution of Volterra integral equation. When it is used?

(v) State and prove change of scale property for Laplace transform.

(vi) Establish the relationship between Fourier transform and Laplace transform.

(vii) Define Boundary value problem.

(viii) Convert $g(x) = 5\cos x + \int_0^x (x-t)g(t)dt$ into a differential equation and find $g(0), g'(0)$.

(ix) Write orthogonal property of Legendre's function.

(x) Show that $J_{\frac{1}{2}}(x) = \left(\sqrt{\frac{2}{\pi x}}\right)\cos x$.

2×10=20

SECTION – B

UNIT - I

2 Solve the following integral equation and discuss all its possible cases

$$g(x) = f(x) + \lambda \int_0^1 (1 - 3xt) g(t) dt .$$

16

OR

2 (a) Show that function $g(x) = \sin\left(\frac{\pi x}{2}\right)$ is a solution of the Fredholm integral equation

$$g(x) - \frac{\pi^2}{4} \int_0^1 K(x,t) g(t) dt = \frac{x}{2} .$$

Where the Kernel is of the form

$$K(x,t) = \begin{cases} \frac{x(2-t)}{2}; 0 \le x \le t \\ \frac{t(2-x)}{2}; t \le x \le 1 \end{cases} .$$

8

(b) Show that the function $g(x) = xe^x$ is a solution of the Volterra integral equation

$$g(x) = \sin x + 2 \int_0^x \cos(x - t) g(t) dt .$$

8

UNIT - II

3 Solve $g(x) = 1 + \lambda \int_0^{\pi} \sin(x+t) g(t) dt$ by the method of Resolvent Kernel. **16**

OR

3 Solve

$$g(x) = 1 - 2x - 4x^2 + \int_0^x \left[3 + 6(x-t) - 4(x-t)^2 \right] g(t) dt$$

by the help of Resolvent Kernel. **16**

UNIT - III

4 (a) If $L[F(t)] = f(p)$ then show that

$$L[t^n F(t)] = (-1)^n \frac{d^n}{dp^n} f(p); n = 1, 2, 3, \ldots\ldots.$$

8

(b) Use Convolution theorem to prove that

$$J_0(t) = \frac{1}{\pi} \int_{-1}^{1} e^{itw} (1 - w^2)^{-1/2} dw.$$

8

OR

4 (a) Find the Fourier transform of

$$f(x)=\begin{cases}1-x^2; |x|<1\\ 0 \quad ; |x|>1\end{cases}$$

and hence evaluate

$$\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx.$$

8

(b) Evaluate $L^{-1}\left[\tan^{-1}\left(\frac{2}{p^2}\right)\right]$.

8

UNIT - IV

5 Solve $ty''+(t-1)y'-y=0, y(0)=5, y'(\infty)$ by help of Laplace transform.

16

OR

5 Solve the following integral equation

$$f(x)\int_0^x \frac{g(t)}{(x-t)^{\alpha}}dt,\ 0<\alpha<1.$$

16

UNIT - V

6 Determine general solution of Hypergeometric equation. **16**

OR

6 (a) Prove that

$$\int_{-1}^{1} x^2 P_{n-1}(x) P_{n-1}(x)dx = \frac{2n(n+1)}{(4n^2-1)(2n+3)}.$$

8

(b) Show that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.

8

https://myqlenet.com