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M.A./M.Sc. (Sem. I) Examination, Dec. 2022

MATHEMATICS

Paper – Math 1C3

(Mechanics)

Time Allowed : Three Hours

Max. Marks : 100

*This question paper contains **three** sections as under :*

Section-A ***Max. Marks-10***

*This section contains one compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. All questions carry equal marks.*

Section-B ***Max. Marks-50***

*This section contains **10** questions having **2** questions from each unit. Answer **5** questions (250 words each) selecting **one** question from each unit. All questions carry equal marks.*

Section-C ***Max. Marks-40***

*This section contains **4** descriptive type questions (questions may have sub-divisions) covering all units but not more than **one** question from each unit. Answer any **two** questions. (**500** words each). All questions carry equal marks.*

Section A

Question No. 1 is compulsory :

1. (i) Find Generalized Potential.

(ii) Explain D' Almbert's principle.

(iii) Define Lagrange Brackets.

(iv) Define Holonomic and non-Holonomic systems.

(v) Write the inertia of elipsoid.

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(vi) Describe Jacobi Integral.

(vii) Derive Lagrangian for an Atwood machine.

(viii) Show that the moment Pi is conserved if $[H, P_i] = 0$ where H-represent Hamiltonian.

(ix) Explain degree of freedom of a system of particles by giving example.

(x) Deduce Newton's second law of motion from Hamilton's principle.

Section B

UNIT-I

2. Show that the transformation :

$P = q \cot p$

$Q = \log \left[\sin \frac{p}{q}\right]$ is canonical and find the generating function.

3. Derive Euler equations of motion for rotating rigid body.

UNIT-II

4. Use the principle of least action. Find the solution of Kepler's problem.

5. State and prove Jacobi-Poisson theorem.

UNIT-III

6. Using Lagrange's and Hamilton's equation of motion, discuss the motion of simple pendulum.

7. Derive the moment of inertia of uniform hemisphere about its axis of symmetry and about an axis lying in the base perpendicular to the system of axis.

UNIT-IV

8. What is the relation between Poisson and Lagrange's Bracket ?

9. Derive Hamilton canonical equations.

UNIT-V

10. (a) Show that if the coordinate corresponding to a rotation is cyclic, the angular momentum is conserved.

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(b) Derive an expression for kinetic energy in terms of generalized coordinates.

11. (a) A smooth straight wire is constrained to rotate in a horizontal plane with a constant angular velocity ω about a vertical axis passing through one end. The point on the wire at a distance b from the axis in the centre of an attractive force proportional to distance. A bead of mass m is free to slide on wire. Find the position of the bead as a function of time.

(b) Derive geodesics in a plane.

Section C

12. Explain with the help of an example the method of separation of variables in Hamilton-Jacobi equation.

13. Explain Lagrangian. Apply Lagrangian formula to explain the motion of bead sliding on a uniformly rotating wire in force free space.

14. (a) Show that the fundamental Poisson Brackets are canonically invariant.

(b) Derive the equation of motion about a fixed point.

(c) Write a short note on Hamiltonian. Write the advantage of Hamilton approach over Newtonian approach.

(d) Analyze the motion of a cylinder rolling down an inclined plane without slipping.

15. If a rectangular parallelelopiped with edges $2a$, $2a$, $2b$ rotates about its centre of gravity under no forces, prove that its angular velocity about one principal axis is constant and about other axis it is periodic.

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