U-5706

M. A. / M. Sc. (Sem. II)
Examination, 2021

MATHEMATICS

PAPER - MATH 2 C(i)

ADVANCED ALGEBRA

Time Allowed : **Three Hours**

Maximum Marks : **70**
Minimum Pass Marks : **28**

Note : *This question paper is divided into three sections as under :*

Section-A ***Max. Marks-10***

This Section contains 01 compulsory question comprising 10 short answer type questions (maximum 20 words answer) taking two questions from each unit. Each question shall be of one mark.

Section-B ***Max. Marks-25***

This Section contains 10 questions, 02 questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question ***from each unit. Each*** *question shall be of five marks.*

Section-C ***Max. Marks-35***

This Section contains four long answer type questions comprising one compulsory question (question no. 12) of 15 marks and four questions of 10 marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in 500 words) and any two more questions (answer about in 400 words) out of remaining four questions.

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SECTION - A

1 (i) Define a kemel of a Homomorphism.

(ii) Define a class equation of a finite group.

(iii) Define a composition series of a group.

(iv) Define a Solvable group.

(v) Define a principal Ideal domain.

(vi) Show that $Z\left[\sqrt{-5}\right] = \left\{a + b\sqrt{-5}\ a, b \in z\right\}$ is not a Euclidean domain.

(vii) Define an unique factorization domain.

(viii) Define a finite extension of field.

(ix) Define an Algebric Extension.

(x) Define a perfect field.

SECTION - B

UNIT - I

2 If N is a normal subgroup of G and $a \in N$, show that every conjugate of a in G is also in N.

3 State Sylow's First theorem. Find all Sylow subgroup of S_3.

UNIT - II

4 Prove that S_n is not solvable for $n \geq 5$.

5 Prove that an infinite abelian group does not have a composition series.

UNIT - III

6 Prove that the ring $(z, +, \square)$ of integers is a principal ideal ring.

7 Prove that Every field is an Euclidean ring.

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UNIT - IV

8 Prove that $J\left[\sqrt{-3}\right]$ is not a U.F.D., J being the ring of integers.

9 If $[k:F]=1$, prove that $k=F$.

UNIT - V

10 If L is an algebric extension of K and K is an algebric extension of F then Prove that L is an algebric extension of F.

11 Prove that every field of characteristic zero is perfect.

SECTION - C

12 List all the conjugate classes in the symmetric group S_3 and verify the class equation.

13 Prove that every Euclidean domain is an unique factorization domain.

14 If N and G/N are solvable then prove that G is solvable.

15 Let K be a finite algebric extension of a field F. Then prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

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