UC-5707

M. A. / M. Sc. (Sem. II) Examination,
June - 2023

MATHEMATICS

Paper - MATH 2C2

Analysis - II

Time Allowed : **Three Hours**

Maximum Marks : **100**

Note : *This question paper contains* ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This section contains ***one compulsory*** *question with* ***10*** *parts, having* ***2*** *parts from each unit, short answer in* ***20*** *words for each part. All questions carry* ***equal*** *marks.*

Section-B ***Max. Marks-50***

This section contains ***10*** *questions having* ***2*** *parts from each unit. Answer* ***5*** *quetsions (250 words each) selecting one question from each unit. All questions carry* ***equal*** *marks.*

Section-C ***Max. Marks-40***

This section contains ***04*** *descriptive type questions (questions may have sub-divisions) covering all units but not more than* ***one*** *question from each unit. Answer* ***two*** *questions. (****500*** *words each). All questions carry* ***equal*** *marks.*

https://pyqsolo.com

SECTION - A

1 (i) Define the upper Riemann-Stieltjes integral.

(ii) Define refinement of a partition.

(iii) Define pointwise convergence of a sequence of function.

(iv) Write the Weierstrass's M-test for uniform convergence.

(v) Define explicit and implicit function.

(vi) Define monotonic function.

(vii) Define Borel measurable function.

(viii) Define outer measure of a set $A \subset R$.

(ix) Define lebesgue integral of the function f over the set E.

(x) Define convergence of sequence of measurable functions.

SECTION - B

UNIT - I

2 Let $f(x)=x, g(x)=x^2$. Does $\int_0^1 f dg$ exist? If it exists, find its value.

OR

Let f be R-integrable function on $[a, b]$ and let g be a monotonically non-decreasing function on $[a, b]$ such that its derivative g' is R-integrable on $[a, b]$, then f is Riemann-Stieltjes integrable on $[a, b]$ and :

$$\int_a^b f dg = \int_a^b f(x) g'(x) dx$$

UNIT - II

3 Show that the series $\sum \frac{(-1)^{n-1}}{n} |x|^n$ is uniformaly convergent on $[-1, 1]$.

OR

Show that the sequence $< f_n >$ where $f_n(x) = nxe^{-nx^2}, x \geq 0$ is not uniformly convergent on $[0, k], k > 0$.

UNIT - III

4 The roots of the equation in λ

$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$$

are u, v, w prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

OR

Show that,

$$f(x, y, z) = (x+y+z)^3 - 3(x+y+z) - 24xyz + a^3$$

has a minima at (1, 1, 1) and a maxima at (−1, −1, −1).

UNIT - IV

5 Prove that the union of two measurable sets is a measurable set.

OR

Show that the function f defined on R by :

$$f(x) = \begin{cases} x+5, & \text{if } x < -1 \\ 2, & \text{if } -1 < x < 0 \\ x^2, & \text{if } x \geq 0 \end{cases}$$

is a measurable function.

UNIT - V

6 Let f be a bounded measurable function on a measurable set E then prove that

$$\left| \int_E f(x)dx \right| \leq \int_E |f(x)|dx$$

OR

Let $< f_n >$ be a sequence of measurable functions converges pointwise to a function f on E, then prove that function f is measurable.

SECTION - C

7 If $f(x)$ is monotonic on $[a, b]$ on $\alpha(x)$ is continuous and monotonic increasing on $[a, b]$, then prove that $f'(x)$ is Riemann-Stieltjes integral with respect to $\alpha(x)$ on $[a, b]$.

8 Test the uniform convergence of the sequence $<f_n>$, where $f_n(x) = nx(1-x)^n$.

9 Prove that the outer measure of an interval is its length.

10 The necessary and sufficient condition for a bounded function f defined on the interval $[a, b]$ to be L-integrable over $[a, b]$ is that for a given $\epsilon > 0$ there exists a measurable partition P of $[a, b]$. Such that

$U(f, p) - L(f, p) < \epsilon$