WA-5707

M. A. / M. Sc. (Sem. - II) Examination, June - 2024

MATHEMATICS

Paper - MATH 2C2

Analysis - II

Time Allowed : **Three Hours**

Maximum Marks : **100**

This question paper contains **two** sections as under:

Section-A **Max. Marks-20**

This section will have 10 questions, having 02 questions from each unit with no internal choice. Answer in about 20 words. All questions will have equal marks. ***(10×02=20)***

Section-B **Max. Marks-80**

This section will have 10 questions, having 02 questions from each unit, one question to be attempted from each unit. Words limit approximately 400-500 words. Questions may have sub divisions. All questions will have equal marks. ***(16×05=80)***

SECTION - A

1 (i) Evaluate $\int_0^2 [x]\,d\left(x^2\right)$.

(ii) What is the improper integral?

(iii) Find the limit function of the sequence $\left\{f_n(x)\right\}$, where $f_n(x)=\dfrac{x^{2n}}{1+x^{2n}}$, $x\in R$.

(iv) Write the statement of Weierstras's M-test.

(v) If $f(x)=x^2+y^2+2x+3y$, then find the directional derivative of $f(x)$ at (2,1) in the direction of the vector $2\hat{i}+\hat{j}$.

(vi) Define function of bounded variation.

(vii) What is the outer measure of the interval $[3, 5)$?

(viii) What is the almost everywhere property?

(ix) Define convergence in measure.

(x) What is the measurable partition?

SECTION - B

UNIT - I

2 (a) Prove that if $f(x)$ is Riemann Stieltjes integrable with respect to $\alpha(x)$ on $[a, b]$. then $|f(x)|$ is also Riemann Stieltjes integrable with respect to $\alpha(x)$ on $[a, b]$ and $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$.

(b) Let $f(x) = x$ and $g(x) = x^2$ be defined on $[0, 1] \to R$. Does $\int_0^1 f d\alpha$ exist? If it is, find its value.

OR

3 (a) Examine the convergence of the integral:

$$\int_0^\infty e^{-ax} \frac{\sin x}{x} dx, a \geq 0$$

(b) Prove that the following integral is convergent : $\int_0^\infty \left(\frac{1}{1+x} - e^{-x} \right) \frac{dx}{x}$.

UNIT - II

4 (a) State and prove Cauchy's criterion for uniform convergence of the sequence of functions.

(b) Show that the sequence $\{f_n(x)\}$, where $f_n(x) = nx\, e^{-nx^2}$, $x \geq 0$ is not uniformly convergent on $[0, K]$, $K > 0$.

OR

5 (a) Test for the series

$$\sum_{n=1}^{\infty} \left[\frac{2n^2x^2}{e^{n^2x^2}} - \frac{2(n-1)^2x^2}{e^{(n-1)^2x^2}} \right]$$ for uniform convergence in [0, 1].

(b) Test the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$ for uniform convergence on $[0, 1]$.

UNIT - III

6 (a) Let $f(x)$ be a function of bounded variation on $[a, b]$. Prove that $f(x)$ is bounded on $[a, b]$, but the converge need not be true.

(b) Let f be a real valued function defined on a domain $D \subset R^2$ and (a, b) be any point of the domain D, such that f_x and f_y are both differentiate at (a, b), then $f_{xy}(a, b) = f_{yx}(a, b)$. prove it.

OR

7 State and prove Implicit function theorem

UNIT - IV

8 (a) Let $\{E_n : n \in N\}$ be a monotonic increasing sequence of measurable sets. Then, prove that

$$m^*\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m^*(E_n)$$

(b) State and prove Countably subadditive property.

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OR

9 (a) Let f and g be two measurable functions defined on a measurable set E. Prove that $f+g$, $f-g$, fg and f/g (if g vanishes no where on the set E) are measurable functions on E.

(b) Prove that a continuous functions $f(x)$ defined on a measurable set E is a measurable function on E, but the converse need not be true.

UNIT - V

10 (a) Let $f(x)$ be a bounded measurable function defined on a measurable set E such that $a \le f(x) \le b$. Prove that $am(E) \le \int_E f(x)\,dx \le b\,m(E)$.

(b) Let $f(x)$ be a bounded function defined on $[a, b]$ and let $f(x)$ be Lebesgue integrable on $[a, b]$. Prove that if $a < c < b$, then $f(x)$ is Lebesgue integrable on $[a, c]$ and $[c, b]$ and

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

OR

11 (a) Let $\{f_n(x)\}$ be a sequence of measurable function is finite a.e. on a set E. Let $\lim_{n\to\infty} f_n(x) = f(x)$ a.e. on E and $f(x)$ be finite a.e. on E. Prove that for each $\sigma > 0$, $\lim_{n\to\infty} m\left[E\left(|f_n - f| \ge \sigma\right)\right] = 0$.

(b) Verify bounded converges theorem for the sequence $\{f_n(x)\}$, where

$$f_n(x) = \frac{1}{\left(1+\frac{x}{n}\right)^n},\ 0 \le x \le 1$$