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**5708**

# M.A./M.Sc. (Sem. II) Examination, 2018

## MATHEMATICS

## PAPER – MATH 2 C(iii)

## PARTIAL DIFFERENTIAL EQUATIONS

**Time Allowed : Three Hours**

**Max. Marks : 70**

**Min. Pass Marks : 28**

*This question paper will be divided into* ***three*** *sections as under :*

***Section-A*** ***Max. Marks-10***

*will carry* ***10*** *marks with* ***1*** *compulsory question comprising* ***10*** *short answer type questions (maximum* ***20*** *words answer) taking* ***two*** *questions from each unit. Each question shall be of* ***one*** *mark.*

***Section-B*** ***Max. Marks-25***

***Will 10 questions, 2 questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question from each unit.***



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***Max. Marks-35***

***Section-C***

*will carry* ***35*** *marks with* ***five*** *long answer type questions comprising* ***one*** *compulsory question (question no.* ***7)*** *of* ***15*** *marks and* ***four*** *questions of* ***10*** *marks each. Students are instructed to attempt total* ***three*** *questions with* ***one*** *compulsory question (answer about in* ***500*** *words) and any* ***two*** *more questions (answer about in* ***400*** *words) out of remaining* ***four*** *questions.*

### Section A

1. (i) Classify the partial differential equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(ii) Define the Envelopes. Give example (only).

(iii) Using the method of separation of variables, solve :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ where } u(x, 0) = 6e^{-3x}$$

(iv) Give the general form of a second order linear partial differential equation in two independent variables $x$ and $y$.



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(vi) What is the D'Alembert's solution of wave equation *i.e.*, the general solution of wave equation $\frac{\partial^2\phi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$.

(vii) Give the Laplace's equation in rectangular Cartesian coordinates in three-dimensional.

(viii) Define wave equation in polar coordinates.

(ix) Give the Laplace's equation in spherical polar coordinates.

(x) Give the Brachistocrone problem.

## Section B

### UNIT-I

2. Give the Lipschitz condition. Use Picard's method to solve the following Initial value problem upto third approximation.

$$\frac{dy}{dx} = 2y - 2x^2 - 3,\ y = 2,\ \text{when } x = 0.$$



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3. Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

### UNIT-II

4. Give the method of separation of variables.

**Solve** $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ by the method of separation of variables subject to the Boundary conditions :

(i) $y = 0$ when $x = 0$

(ii) $y = 0$ when $x = l$

(iii) $\frac{\partial y}{\partial t} = 0$ when $t = 0$

(iv) $y = \begin{cases} \frac{2k}{l}x, & 0 < x < \frac{l}{2} \\ \frac{2k}{l}(l-x), & \frac{l}{2} < x < l \end{cases}$ when $t = 0$

5. State and prove Cauchy problem.

### UNIT-III

6. State and prove the D'Alembert's solution of wave equation for Cartesian coordinates.



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7. Find a solution of the equation

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \phi}{\partial \theta}\right) = 0$$

of the form $\phi = f(r)\cos\theta$, given that $\frac{-\partial\phi}{\partial r} = V\cos\theta$, when $r = a$ and $\frac{-\partial\phi}{\partial r} = 0$ when $r = \infty$.

## UNIT-IV

8. Show that in spherical polar coordinate $r$, $\theta$, $\phi$, Laplace equation possesses solution of the form

$$\left(Ar^n + \frac{B}{r^{n+1}}\right) F(\cos\theta) e^{\pm im\phi}$$

where A, B, $m$ and $n$ are constants and $F(\mu)$ satisfies the differential equation :

$$(1-\mu^2)\frac{d^2E}{d\mu^2} - 2\mu\frac{dF}{d\mu} + \left\{n(n+1) - \frac{m^2}{(1-\mu^2)}\right\}F = 0.$$

9. By the method of separation of variables, solve the Laplace's equation in spherical polar coordiantes.



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## UNIT-V

10. Define Functionals. State and prove Euler's differential equation for an extremal.

11. A particle is moving with a force perpendicular to its distance from the line of zero velocity. Show that the path of quickest descent is a circle.

## Section C

12. (a) Show that the geodesics on a sphere of radius $a$ are its great circles.

(b) Find the curve joining the points $(x_1, y_1)$ and $(x_2, y_2)$ that yields a surface of revolution of minimum area when revolved about the axis. Also find the extremal of the functional.

13. (a) Define the types of Hyperbolic, elliptic and parabolic partial differential equations. Classify the following partial differential equations as to type in $xy$-planes

$$x^2y\frac{\partial^2 z}{\partial x^2} + xy\frac{\partial^2 z}{\partial y \partial x} - y^2\frac{\partial^2 z}{\partial y^2} = 0.$$



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(b) Give the solution of one-dimensional Heat equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ where $z = z(x, t)$ with the conditions that $z = 0$, when $x = 0$ and $x = l$ for all values of $t$ i.e., with Boundary conditions $z(0, t) = z(l, t) = 0$ for all $t$

14. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$.

It is released from rest this position. Find the displacement $y(x, t)$.

(b) Find the extremals for the integrals defined by $I = \int_0^1 (y^2 + y'^2 + 2ye^x)\, dx$ passing through the points (0, 0) and (1, $e$).

15. (a) Reduce the equation

$$(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = ny^{2n-1} \frac{\partial z}{\partial y}$$

to canonical form and find its general solution.



(b) A thin rectangular plate whose surface is impervious to heat flow has at $t = 0$ and arbitrary distribution of temperature $f(x, y)$. Its four edges $x = 0$, $x = a$, $y = 0$ and $y = b$ are kept at zero temperature. Determine the temperature at a point of the plate as $t$ increases.

16. State the existence and uniqueness theorem for solution of the equation :

$$\frac{dy}{dx} = f(x, y).$$ (Statement only)

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