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5708

M.A./M.Sc. (Second Semester)

Examination, 2019

MATHEMATICS

Paper MATH-2 C(iii)

(Partial Differential Equations)

Time Allowed : Three Hours

Maximum Marks : 70

Min. Pass Marks : 28

Note : — This question paper will be divided into ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This Section contains ***1*** *compulsory question comprising* ***10*** *short answer type questions (maximum* ***20*** *words answer) taking two questions from each unit. Each question shall be of one mark.*

Section-B ***Max. Marks-25***

This Section contains ***10*** *questions,* ***2*** *questions from each unit (answer in about* ***250*** *words). Students are instructed to attempt* ***five*** *questions by selecting* ***one*** *question from each unit. Each question shall be of five mark.*

Section-C ***Max. Marks-35***

This Section contains five long answer type questions comprising one compulsory question (question No. ***12****) of* ***15*** *marks and four questions of* ***10*** *marks each. Students are instructed to attempt total* ***three*** *questions with one compulsory question (answer in about* ***500*** *words) and any* ***two*** *more questions (answer in about* ***400*** *words) out of remaining four questions.*

Section A

1. (i) Classify the following partial differential equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial z^2}$$

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(ii) Give Lipschitz condition for a function $f(x, y)$ in a region D in x-y plane.

(iii) Define principle of superposition in the method of separation of variable.

(iv) Define Cauchy problem.

(v) Write down *three*-dimensional wave equation in Cartesian co-ordinates system.

(vi) Write the heat equation in two dimension in Cartesian co-ordinate system, if the heat flow is steady.

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(vii) Write two-dimensional Laplace's equation in polar co-ordinates.

(viii) Give heat equation in cylindrical co-ordinates.

(ix) Define Functional.

(x) Define Isoperimetric problems.

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Section B

UNIT-I

2. Examine the existence and uniqueness of solution of the following IVP :

$$\frac{dy}{dx} = \sqrt{|y|} \quad y(0) = 0.$$

3. Reduce the following equation in canonical form :

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$$

UNIT-II

4. Solve the boundary value problem :

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, \text{ if } u(0, y) = 8e^{-3y} + 4e^{-5y}.$$

5. Solve the Cauchy problem for P.D.E. :

$$2p + 3q = 1$$

with Cauchy data $bx - ay = 0$, $z = f\left(\frac{x}{a}\right)$ where a and b are constants and $f\left(\frac{x}{a}\right)$ is given function, assuming that $3a - 2b \neq 0$. Discuss the solution when $3a = 2b$.

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UNIT-III

6. Solve the heat equation :

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t};\ 0 \le x \le 2\pi$$

subject to boundary condition $u(x, 0) = \sin^3 x$ and $u(0, t) = u(2\pi, t) = 0$ for $t \ge 0$.

7. A tightly stretched string with fixed end points $x = 0$ and $x = l$ initially in a position given by :

$$y = y_0 \sin^3 \frac{\pi x}{l},$$

it is released from rest from this position. Find $y(x, t)$.

UNIT-IV

8. Solve the differential equation :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

subject to boundary conditions :

(*i*) u is finite when $r \to 0$

(*ii*) $u = \sum C_n \cos n\theta$ when $r = a$

9. Solve three-dimensional wave equation in spherical polar co-ordinates.

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UNIT-V

10. Find the path on which a particle, in the absence of friction, will slide from one fixed point to another point in the shortest time under the action of gravity.

11. Prove that the sphere is solid figure of revolution, which for a given surface area has maximum volume.

Section C

12. (*a*) Find the shortest curves joining two points (x_1, y_1) and (x_2, y_2).

(*b*) Show that the geodesics on a sphere of radius a are its great circles.

13. If u is a function of r and θ satisfying :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

within the region $r = a, r = b, \theta = 0$ and $\theta = \frac{\pi}{2}$, its value along the boundary $r = a$ is $\theta\left(\frac{\pi}{2} - \theta\right)$ and its value along the other boundary is zero, prove :

$$u = \frac{2}{\pi}\sum \frac{\left(\frac{r}{b}\right)^{4m-2} - \left(\frac{b}{r}\right)^{4m-2}}{\left(\frac{a}{b}\right)^{4m-2} - \left(\frac{b}{a}\right)^{4m-2}} \frac{\sin(4m-2)\theta}{(2m-1)^3}.$$

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14. State existence and uniqueness theorem for the solution of the equation :

$$\frac{dy}{dx} = f(x, y);\ y(x_0) = y_0.$$

Give the proof for existence of the solution only.

15. (a) A thin rectangular plate whose surface is impervious to heat flow has at $t = 0$ on arbitrary distribution of temperature $f(x, y)$. Its four edges :

$$x = 0,\ x = a,\ y = 0,\ y = b$$

are kept at zero temperature. Determine the temperature at a point of the plate as t increases.

(b) Reduce the equation in canonical form :

$$(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = n y^{2n-1} \frac{\partial z}{\partial y}$$

and find its general solution.

16. (a) Show that the general solution of one-dimensional wave equation :

$$c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

is $u(x, t) = \phi(x - ct) + \Psi(x - ct)$.

(b) Find the extremals of the functionals :

$$I\left[y(x), z(x)\right] = \int_0^{\frac{\pi}{2}} \left[2yz + (y')^2 + (z')^2\right] dx$$

given $y(0) = 0,\ y\left(\frac{\pi}{2}\right) = -1,$

$z(0) = 0,\ z\left(\frac{\pi}{2}\right) = 1.$

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