U-5708

M. A. / M. Sc. (Sem. II)
Examination, 2021

MATHEMATICS

PAPER - MATH 2 C (iii)

PARTIAL DIFFERENTIAL EQUATIONS

Time allowed : **Three Hours**

Maximum Marks : **70**
Min. Pass Marks : **28**

Note : *This question paper will be divided into* ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This section contains 01 compulsory question comprising 10 short answer type questions (maximum 20 words answer) taking two questions from each unit. Each question shall be of one mark.

Section-B ***Max. Marks-25***

This section contains 10 questions, 02 questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question from each unit. Each question shall be of five marks.

Section-C ***Max. Marks-35***

This section contains four long answer type questions comprising one compulsory question (question no. 12) of 15 marks and four questions of 10 marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in 500 words) and any two more questions (answer about in 400 words) out of remaining four questions.

https://pyqlab.com

SECTION - A

1 (i) Classify the following partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial^2 u}{\partial y^2}=0$.

(ii) Define the envelopes. Give example.

(iii) Using the method of separation of variables solve $\frac{\partial^2 u}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$.

(iv) Give the general form of a second order linear partial differential equation in two independent variables x and y.

(v) What is the D' Alembert's solution of wave equation. i.e. the general solution of wave equation $\frac{\partial^2 \phi}{\partial x^2}=+\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}$.

(vi) Write the heat equation in two dimension in cartesian co-ordinates system. if the heat flow is steady.

(vii) Give the Laplace's equation in spherical polar coordinates.

(viii) Give heat equation in cylindrical coordinates.

(ix) Define the Geodesic, give example.

(x) Define Isoperimetric problem.

SECTION - B

UNIT - I

2 Give the Lipschitz condition, use Picard method to solve the following Initial value problem upto third approximation.

$\frac{dy}{dx}=2y-2x^2-3,\ y=2$ when $x=0$.

3 Find the complete integral of :

$(y-x)(q\,y-px)=(p-q)^2$

UNIT - II

4 Reduce the equation

$$y^2\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x \partial y}+x^2\frac{\partial^2 z}{\partial y^2}=\frac{y^2}{x}\frac{\partial z}{\partial x}+\frac{x^2}{y}\frac{\partial z}{\partial y}$$

to canonical form and hence solve it.

5 Use the method of separation of varibles to solve the following partial differential equations :

$$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$$

UNIT - III

6 Find a solution of the equation $\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \phi}{\partial \theta}\right)=0$ of the form $\phi = f(r)\cos\theta$, given that $-\frac{\partial \phi}{\partial r} = v\cos\theta$, where $r = a$ and $-\frac{\partial \phi}{\partial r} = 0$ when $r = \infty$.

7 A tightly stretched string with fixed end points $x = 0$ and $x = l$ initially in a position given by : $y = y_0 \sin^3 \frac{\pi x}{l}$, it is released from rest from this position find $y(x, t)$.

UNIT - IV

8 Show that in spherical polar coordinate r, θ, ϕ Laplace equation possesses solution of the form $\left(A r^n + \frac{B}{r^{n+1}}\right) F(\cos\theta)\, e^{\pm im\phi}$ where A, B, m and n are constants and $F(\mu)$ satisfies the differential equation

$$\left(1-\mu^2\right)\frac{d^2F}{d\mu^2} - 2\mu\frac{dF}{d\mu} + \left\{n(n+1) - \frac{m^2}{\left(1-\mu^2\right)}\right\}F$$

9 Solve the differential equation :

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0$$

Subject to the boundary conditions :

(i) V is finite when $r \to 0$.

(ii) $V = \sum c_n \cos n\theta$. when $r = a$.

UNIT - V

10 A particle is moving with a force $\perp$ perpendicular to and proportional to its distance from the line of zero velocity. Show that the path of quickest descent is a circles.

11 Find the path on which a particle, in the absence of friction, will slide from one fixed point to another point in the shortest time under the action of gravity.

SECTION - C

12 (a) If u is a function of r and θ satisfying $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ with in the region $r = a$, $r = b$, $\theta = 0$ and $\theta = \frac{\pi}{2}$, its

value along the boundary $r = a$ is $\theta\left(\frac{\pi}{2} - \theta\right)$ and its value along the other boundary is zero, prove :

$$u = \frac{2}{\pi}\sum \frac{\left(\frac{r}{b}\right)^{um-2} - \left(\frac{b}{r}\right)^{um-2}}{\left(\frac{a}{b}\right)^{um-2} - \left(\frac{b}{a}\right)^{um-2}} \times \frac{\sin(um-2)\theta}{(2m-1)^3}$$

(b) The second order partial differential equation

$$\left(1-\sqrt{xy}\right)\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \left(1+\sqrt{xy}\right)\frac{\partial^2 u}{\partial y^2} = 0$$

(i) Hyperbolic in the second and fourth quadrants.

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(ii) Elliptic in the first and the third quadrants.

(iii) Hyperbolic in the second and elliptic in the fourth quadrants.

(iv) Hyperbolic in the first and the third quadrant.

13 (a) A thin rectangular plate whose surface is impervious to heat flow has at $t = 0$ an arbitary distribution of temperature $f(x, y)$. Its four edges :

$x = 0, x = a, y = 0, y = b$

are kept at zero temperature. Determine the temperature. Determine the temperature at a point of the plate as t increases.

(b) Reduce the equation in canonical form:

$$(n-1)^2\frac{\partial^2 z}{\partial x^2} - y^{2n}\frac{\partial^2 z}{\partial y^2} = n\,y^{2n-1}\frac{\partial z}{\partial y}$$

and find its general solution.

14 (a) By the method of separation of variables, solve the Laplace's equation in spherical polar coordinates :

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 u}{\sin^2\theta \partial \phi^2}$$

(b) Show that the geodesics on a sphere of radius a are its great circles.

15 (a) State and prove the existence and uniqueness theorem for solution of the equation : $\frac{dy}{dx} = f(x, y)$.

(b) Find the shortest curve joining two points (x_1, y_1) and (x_2, y_2).