UC-5708

M. A. / M. Sc. (Sem. II) Examination, June 2023

MATHEMATICS

Paper - MATH 2C3

Partial Differential Equations

Time Allowed : **Three Hours**

Max. Marks : **100**

Note : This question paper contains three sections as under :

Section-A ***Max. Marks-10***

This section contains one compulsory question with 10 parts, having 2 parts from each unit, short answer in 20 words for each part. All questions carry equal marks.

Section-B ***Max. Marks-50***

This section contains 10 questions having 2 questions from each unit. Answer 5 questions (250 words each) selecting one question from each unit. All questions carry equal marks.

Section-C ***Max. Marks-40***

This section contains 04 descriptive type questions (questions may have sub-division) covering all units but not more than one question from each unit. Answer any two questions. (500 words each). All questions carry equal marks.

https://pyqlet.com

SECTION-A

1 (i) Define initial value problem.

(ii) Give the statement of Lipschitz condition.

(iii) Define Elliptic linear P.D.E. of second order.

(iv) Write the equations of Heat and Wave equations in cartesian coordinates.

(v) Write Sturm-Liouville Boundary value problem with conditions.

(vi) Define Rayleigh Quotient.

(vii) Define functionals in calculus of variation.

(viii) Define isoperimetric problem.

(ix) Write the equations of Euler's for n functions.

(x) Define Geodesic.

SECTION-B

UNIT-I

2 Solve the IVP $y' = x+y$, $y=1$ when $x = 0$ by Picard's method and compare the solution with the exact solution.

OR

3 Examine the existence and uniqueness of solution of IVP

$$\frac{dy}{dx} = 3y^{2/3},\ y(0) = 0.$$

UNIT-II

4 Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$ into canonical form and hence solve it.

OR

5 Using the method of separation of variables to solve the following PDE :

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

UNIT-III

6 Find the eigen values and eigen functions of the Sturm-Liouville problem.

$$y'' + \lambda y = 0;\ y(0) = 0,\ y'\left(\frac{\pi}{2}\right) = 0$$

OR

https://pyqlelo.com

7 Solve three dimensional wave equation in spherical polar coordinates:

$$\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial \varphi^2}$$

$$=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

UNIT-IV

8 Find the shortest curves joining two points (x_1, y_1) and (x_2, y_2).

OR

9 Find the extremals of the functionals

$$I\left[y(x), z(x)\right]=\int_0^{\pi/2}\left[2yz+(y')^2+(z')^2\right]dx$$

given that

$$y(0)=0,\ y\left(\frac{\pi}{2}\right)=-1,\ z(0)=0,\ z\left(\pi/2\right)=1.$$

UNIT-V

10 Prove that the extremals of

$$I=\int_{x_1}^{x_2}\left[p(x)\cdot y'^2-q(x)\cdot y^2\right]dx$$

subject to the constraints

$$J=\int_{x_1}^{x_2} r(x)\cdot y^2 dx=1$$

are the solutions of the Sturm-Liouville equation

$$\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right]+\left[q(x)+\lambda r(x)\right]y=0.$$

OR

11 Find the curve of fixed length L that joins the points (0,0) and (1,0) lies above the x-axis, and encloses the maximum area between itself and the x-axis.

SECTION-C

12 Give the statement of Existence and uniqueness theorem for the solution of the equation $\frac{dy}{dx}=f(x, y)$. Also, obtain the third approximation to the following IVP :

$$\frac{d^2y}{dx^2} = x^2\frac{dy}{dx} + x^4y;\ y = 5,\ \frac{dy}{dx} = 1 \text{ at } x = 0.$$

using Picard's method.

13 Solve the two-dimensional Heat equation in Cartesian form

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{K}\frac{\partial u}{\partial t}.$$

14 Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ satisfying the BC's :

$u(0, y) = u(l, y) = u(x, 0) = 0$ and

$$u(x, a) = \sin\left(\frac{n\pi x}{l}\right).$$

15 Derive Euler's differential equation for an extremal under the simplest variational problem.

https://pyqlelo.com