WA-5708

M. A. / M. Sc. (Sem. - II) Examination, June - 2024

MATHEMATICS

Paper - MATH 2C3

Partial Differential Equations

Time Allowed : **Three Hours**

Maximum Marks : **100**

This question paper contains **two** sections as under:

https://rvqelo.com

Section-A ***Max. Marks-20***

*This section will have 10 questions, having **02** questions from each unit with no internal choice. Answer in about **20** words. All questions will have equal marks.* ***(10×02=20)***

Section-B ***Max. Marks-80***

*This section will have **10** questions, having **02** questions from each unit, one question to be attempted from each unit. Words limit approximately **400-500** words. Questions may have sub divisions. All questions will have equal marks.* ***(16×05=80)***

https://pyqseo.com

SECTION - A

1 (i) State Lipschitz condition.

(ii) Write the equation of Heat equation in three dimensional co-ordinates.

(iii) Classify the following DE :

$$\sqrt{y^2+x^2}\,u_{xx}+2(x-y)\cdot u_{xy}+\sqrt{y^2+x^2}\cdot u_{yy}=0$$

(iv) Define Cauchy's Problem.

(v) Write three dimensional wave equation in polar form.

(vi) Define Jacobian of the transformation.

(vii) Define Functional.

(viii) Write Euler's differential equation for an extremal.

(ix) Define Geodesics.

(x) Define Isoperimetric problem.

SECTION - B

UNIT - I

2 (a) Use Taylor's series to find approximate solution of $\frac{dy}{dx} = 3x + y^2; y(0) = 1$ at $x = 0.1$.

(b) Solve the IVP : $y' = x + y; y(0) = 1$ by Picard's method and compare the solution with exact solution.

OR

(a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial yz} = 0$ into Canonical form and hence solve it.

(b) Solve $\left(p^2 + q^2\right)y = qz$ by charpit's method.

UNIT - II

3 Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ satisfying the BC's :

$u(0, y)=u(l, y)=u(x, 0)=0$ and

$u(x, a)=\sin\left(\frac{n\pi x}{l}\right)$.

OR

Solve the problem

$xu_x+\left(x^2+y\right)u_y+\left(\frac{y}{x}-x\right)u=1$ with the IC that $u(1,1)=0$. Also, define characteristics.

UNIT - III

4 Derive D'Alembert's solution of wave equation

$$\frac{\partial^2 \phi}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}.$$

OR

Solve $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ by the method of separation of variables subject to the BC's :

(i) $y = 0$ at $x = 0$

(ii) $y = 0$ at $x = l$

(iii) $\frac{\partial y}{\partial t} = 0$ at $t = 0$

(iv) $y = \begin{cases} \frac{2kx}{l}, 0 < x < l/2 \\ \frac{2k}{l}(l - x), \frac{l}{2} < x < l \text{ when } t = 0 \end{cases}$

UNIT - IV

5 (a) Find the extremal for the functional $\int_1^2 \frac{\sqrt{1+(y')^2}}{x} dx, y(1) = 0, y(2) = 1.$

(b) Find the shortest curves joining two points (x_1, y_1) and (x_2, y_2).

OR

Derive Euler-Lagrange's equation for an extremal.

UNIT - V

6 (a) A particle is moving with a force perpendicular to and proportional to its distance from the line of zero velocity. Show that the path of quickest descent is a circle.

(b) Show that the geodesics on a plane are straight lines.

OR

(a) Prove that the sphere is the solid figure of revolution which, for a given surface area, has maximum volume.

(b) Find the extremals of the functional $I[y(x)] = \int_0^1 \left(y'^2 - y^2\right) dx$ subject to the following BC : https://www.uokonline.com

(i) $y(0) = 0, y(1) = 1$

(ii) $y(0) = 0$

(iii) $y(1) = 1$

(iv) No end conditions are specified.