This question paper contains 4+2 printed pages.

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M.A./M.Sc. (Sem. II) Examination, 2018

MATHEMATICS

Paper – Math 2C(ii)

(Real Analysis)

Time Allowed : Three Hours

Max. Marks : 70

Min. Pass Marks : 28

This question paper will be divided into ***three*** *sections as under :*

Section-A ***Max. Marks-10***

*Will carry **10** marks with **1** compulsory question comprising **10** short answer type questions (maximum **20** words answer) taking two questions from each unit. Each question shall be of one mark.*

Section-B ***Max. Marks-25***

*Will **10** questions, **2** questions from each unit (answer about in **250** words). Students are instructed to attempt five questions by selecting **one question** from each unit.*

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Max. Marks-35

Section-C

*Will carry **35** marks with five long answer type questions comprising one compulsory question (question no. **7**) of **15** marks and four questions of **10** marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in **500** words) and any two more questions (answer about in **400** words) out of remaining four questions.*

Section A

1. (i) Write the necessary and sufficient condition for a function to be Riemann-Stieltjes integrable.

(ii) Define the upper Riemann-Stieltjes integral.

(iii) Define pointwise convergence of a sequence of functions.

(iv) Write the Weierstrass's M-test for uniform convergence.

(v) Define outer measure of a set $A \subset R$.

(vi) Find the Lebesgue measure of the set of integers.

(vii) Define a Lebesgue measurable function.

(viii) Define equivalent functions on the same set.

(ix) What do you understand by the convergence of a sequence of measurable functions ?

(x) Define Lebesgue integral of the function f over the set E.

Section B

UNIT-I

2. Evaluate $\int_0^3 x \, d[x]$, where $[x]$ is the greatest integer not exceeding x.

Or

Let $f(x) = K$ be a constant function defined on $[a, b]$ and $\alpha(x)$ is a monotonically non-decreasing function on $[a, b]$. Then prove that $f(x)$ is Riemann - Stieltjes integrable.

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3. Show that the series $\sum \frac{(-1)^{n-1}}{n} |x|^n$ is uniformly convergent in $[-1, 1]$.

Or

If $\langle f_n \rangle$ is a sequence of continuous functions on an interval $[a, b]$ and if f_n converges uniformly to f on $[a, b]$, then prove that f is continuous on $[a, b]$.

UNIT-III

4. Prove that the outer measure of a set is translation invariant.

Or

If E is a countable set, then prove that

$$m^*(E) = 0.$$

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UNIT-IV

5. Show that a constant function defined on a measurable set is measurable.

Or

Let f and g be measurable functions defined on a measurable set E. Then prove that $f + g$ is measurable. http://www.uokonline.com

UNIT-V

6. Let f be a bounded measurable function on a measurable set E. then prove that

$$\left| \int_E f(x)\,dx \right| \leq \int_E |f(x)|\,dx$$

Or

Let f be a bounded measurable function on a set E and let $f(x) \geq 0$ almost everywhere on E. If $\int_E f(x)\,dx = 0$, then prove that $f(x) = 0$ almost everywhere on E.

[Contd....

Section C

7. If $f(x)$ is monotonic on $[a, b]$ on $\alpha(x)$ is continuous and monotonic increasing on $[a, b]$, then prove that $f(x)$ is Riemann-Stieltjes integrable with respect to $\alpha(x)$ on $[a, b]$.

8. Test for uniform convergence of the sequence $\langle f_n \rangle$, where $f_n(x) = nx(1 - x)^n$.

9. Prove that every interval is measurable.

10. Prove that there exists a non-measurable set.

11. Let a sequence $\langle f_n \rangle$ of measurable functions defined as a measurable set E converge pointwise to a function f on E. Then prove that the function f is measurable.

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