This question paper contains 8 printed pages]

5707

M.A./M.Sc. (II Sem.)

Examination, 2019

MATHEMATICS

Paper-Math 2C(*ii*)

(Real Analysis)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 28

This question paper will be divided into ***three*** *sections as under :*

Section-A ***Max. Marks-10***

Will carry ***10*** *marks with* ***1*** *compulsory question comprising* ***10*** *short answer type questions (maximum* ***20*** *words answer) taking* ***two*** *questions from each unit.* ***Each*** *question shall be of* ***one*** *mark.*

Section-B ***Max. Marks-25***

Will carry ***10*** *questions,* ***2*** *questions from each unit (answer in about* ***250*** *words). Students are instructed to attempt* ***five*** *questions by selecting* ***one*** *question from each unit.*

Section-C ***Max. Marks-35***

Will carry ***35*** *marks with* ***five*** *long answer type questions comprising* ***one*** *compulsory question (question No.* ***7)*** *of* ***15*** *marks and* ***four*** *questions of* ***10*** *marks each. Students are instructed to attempt total* ***three questions with one*** *compulsory question (answer in about* ***500*** *words) and any* ***two*** *more questions (answer in about* ***400*** *words) out of remaining* ***four*** *questions.*

Section A

1. (*i*) Define refinement of a partition.

   (*ii*) Define Riemann-Stieltjes integral.

   (*iii*) Define pointwise convergence of a sequence of functions.

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(iv) Find the limit function of the sequence $\langle f_n(x) = nx(1 - x)^n \rangle$.

(v) Define Lebesgue outer measure.

(vi) Define Borel set.

(vii) Define measurable function.

(viii) Define limit superior.

(ix) Write the definition of convergence almost everywhere of a sequence of measurable functions.

(x) Define Lebesgue integral of a bounded function.

Section B

UNIT-I

2. Let $f(x) = x$, $g(x) = x^2$. Does $\int_0^1 f\,dg$ exist ? If it exists, find its value.

Or

Let f be R-integrable function on $[a, b]$ and let g be a monotonically non-decreasing function on $[a, b]$ such that its derivative g' is R-integrable on $[a, b]$, then f is Riemann-Stieltjes integrable on $[a, b]$ and :

$$\int_a^b f\,dg = \int_a^b f(x)\,g'(x)\,dx.$$

UNIT-II

3. Let $\langle f_n \rangle$ be a sequence of functions defined on a set $[a, b]$. Let $\lim_{n\to\infty} f_n(x) = f(x)$ for all $x \in [a, b]$ and let

$$M_n = \sup\left[f_n(x) - f(x)\right] \text{ s.t. } x \in [a, b]$$

Then $\langle f_n \rangle$ converges uniformly to f if and only if :

$$M_n \to 0 \text{ as } n \to \infty.$$

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Or

Test for uniform convergence of the sequence and continuity of the limit function f :

$$< f_n(x) = n^2 x(1-x)^n \mid x \in \mathrm{R} > \text{ and } \forall\ n \in \mathrm{N}.$$

UNIT-III

4. Let $\{E_n : n \in N\}$ be a sequence of disjoint measurable sets, then prove that :

$$m^*\left[\bigcup_{i=1}^{\infty} E_i\right] = \sum_{i=1}^{\infty} m^*(E_i).$$

Or

Let $\{E_n\}$ be a decreasing sequence of measurable sets and $m(E_n) < \infty$ for each n. Then prove that :

$$m\left[\bigcap_{n=1}^{\infty} E_n\right] = \lim_{n \to \infty} m(E_n).$$

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UNIT-IV

5. Let f and g be measurable functions defined on a measurable set E, then $f + g$ and fg are measurable functions on E.

Or

Show that the function f defined on R by :

$$f(x) = \begin{cases} x+5, & \text{if } x < -1 \\ 2, & \text{if } -1 < x < 0 \\ x^2, & \text{if } 0 \le x \end{cases}$$

is a measurable function.

UNIT-V

6. Let $< f_n >$ be a sequence of measurable functions converges pointwise to a function f on E, then prove that function f is measurable.

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Or

Every bounded measurable function f on a measurable set E is Lebesgue integrable on E.

Section C

7. Let f be a bounded function and g be a monotonically non-decreasing function on $[a, b]$. Then $f \in RS(g)$ if and only if for every $\epsilon > 0$ there exists a partition P of $[a, b]$ such that :

$$U(P, f, g) - L(P, f, g) < \epsilon.$$

8. Show that the sum of the series :

$$\sum\left(\frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2}\right)$$

is continuous for all values of x, although O is a point of non-uniform convergence of the series.

9. Prove that the family M of all measurable sets is a σ algebra of sets in P(R).

10. Prove that :

(*i*) A Borel measurable set is measurable.

(*ii*) If f and g are real-valued functions defined on a measurable space X, then $f \cup g$ and $f \cap g$ are measurable functions.

11. If f is bounded function defined on $[a, b]$ and f is R integrable on $[a, b]$, then f is also L-integrable on $[a, b]$ and $L\int_a^b f(x)\,dx = R\int_a^b f(x)\,dx$.

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