U-5707

M. A. / M. Sc. (Sem.-II) Examination, 2021

MATHEMATICS

PAPER - MATH 2 C(ii)

Real Analysis

Time allowed : **Three Hours**

Maximum Marks : **70 (Min. Pass Marks : 28)**

Note : This question paper will be divided into three sections as under :

Section-A ***Max. Marks-10***

*This section contains **01 compulsory** question comprising 10 short answer type questions (maximum 20 words answer) taking two questions from each unit. Each question shall be of one marks.*

Section-B ***Max. Marks-25***

*This section contains **10** questions, (02 questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question from each unit. Each question shall be of five mark.*

Section-C ***Max. Marks-35***

This section contains five long answer type questions comprising one compulsory question (question no. 07) of 15 marks and four questions of 10 marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in 500 words) and any two more questions (answer about in 400 words) out of remaining four questions.

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SECTION A

1 (i) Define Riemann-Stieltjes sums.

(ii) Write the necessary and sufficient condition for a function to be Riemann-Stieltjes integrable.

(iii) Write the Weierstrass's M-test for uniform convergence.

(iv) Define uniform convergence of sequence.

(v) Prove that R is a measurable set.

(vi) Define outer measure of a set A C R.

(vii) Define measurable function.

(viii) Define Borel measurable function.

(ix) Define Upper Lebesgue integral.

(x) What do you understand by the convergence of a sequence of measurable functions ?

SECTION B

UNIT I

2 If f(x) = K be a constant function defined on [a, b] and $\alpha(x)$ is a monotonically non decreasing function on [a, b], then prove that f(x) is Riemann-Stieltjes integrable.

OR

If f be Riemann-Stieltjes integrable with respect to $\alpha(x)$ on [a, b] then prove that

$$m[\alpha(b)-\alpha(a)] \le \int_a^b f\, d\alpha \le M[\alpha(b)-\alpha(a)]$$

where

$m = \inf_{x \in [a,b]} f(x)$, $M = \sup_{x \in [a,b]} f(x)$

UNIT II

3 Show that the series $\sum \frac{(-1)^{n-1}}{n} |x|^n$ is uniformly convergent on [−1, 1]

OR

Show that the sequence $< f_n >$ where

$$f_n(x) = nxe^{-nx^2}, \quad x \ge 0$$

is not uniformly convergent on [o, k], k > 0

UNIT III

4 Prove that the union of two measurable sets is a measurable set.

OR

If $\{E_n\}$ be a countable collection of sets real numbers then

$$m^*\left(\bigcup_n E_n\right) \leq \sum_n m^*(E_n)$$

UNIT IV

5 Prove that the characteristic function ϕ_A of a set A is measurable if and only if A is a measurable set.

OR

Prove that a continuous function defined on a measurable set is always measurable.

UNIT V

6 If $< f_n >$ is a sequence of measurable functions defined on a measurable set E that converges in measure to the function f defined on E, then $< f_n >$ converges in measure to every function g which is equivalent to f.

OR

Let f be a bounded measurable function on a measurable set E then prove that

$$\left|\int_E f(x)\,dx\right| \leq \int_E |f(x)|\,dx$$

SECTION C

7 If f_1 and f_2 are Riemann-Stieltjes integrable with respect to $\alpha(x)$ on [a, b]. Then f_1+f_2 is Riemann-Stieltjes integrable with respect to $\alpha(x)$ on [a, b] and

$$\int_a^b (f_1+f_2)\,d\alpha = \int_a^b f_1\,d\alpha + \int_a^b f_2\,d\alpha$$

8 Show that the sum of the series

$$\sum\left|\frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2}\right|$$

is continuous for all values of x, although O is a point of non-uniform converges of the series.

9 Prove that the outer measure of an interval is its length.

10 Let f be a measurable function, finite almost everywhere defined on the closed interval E = [a, b]. Then for all $\sigma > 0$ and $\in > 0$ there exists a continuous function ϕ defined on E such that

$$(\{x \in E / |f(x)-\phi(x)| \geq \sigma\}) < \in$$

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11 The necessary and sufficient condition for a bounded function f defined on the interval [a, b] to be L-integrable over [a, b] is that for a given $\in > 0$ there exists a measurable partition P of [a, b] such that
$U(f, p) - L(f, p) < \in$.

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