U-5709

M. A. / M. Sc. (Sem. II) Examination, 2021

Mathematics

PAPER - MATH 2 C(iv)

SPECIAL FUNCTION

Time allowed : **Three Hours**

Maximum Marks : **70**
Min. Pass Marks : **28**

Note : *This question paper will be divided in to* ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This section contains 01 compulsory question comprising 10 short answer type questions (maximum 20 words answer) taking two questions from each unit. Each question shall be of one mark.

Section-B ***Max. Marks-25***

This section contains 10 questions, 02 questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question from each unit. Each question shall be of five marks.

Section-C ***Max. Marks-35***

This section contains five long answer type questions comprising one compulsory question (question no. 07) of 15 marks and four questions of 10 marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in 500 words) and any two more questions (answer about in 400 words) out of remaining four questions.

https://www.uokonline.com

SECTION - A

1 (i) Define the notation $(\alpha)_n$.

(ii) Write absolute convergence of $F(a, b; C; Z)$.

(iii) Define exponential function as special case of ${}_pF_q$.

(iv) Define contiguous function relation for ${}_pF_q$.

(v) Write Bessel's differential equation.

(vi) Define $J_n(x)$ in terms of ${}_0F_1$.

(vii) Write the Kummer's First formula.

(viii) Write Barne's type integral representation of ${}_1F_1$.

(ix) Define generating function for $F(x, t)$.

(x) If $G(2xt - t^2) = \sum_{n=0}^{\infty} g_n(x)t^n$, $g_0(x) = 0$ then write the differential recurrence relation for $g_n(x)$ for $n \geq 1$.

SECTION - B

UNIT - I

2 State and prove Gauss's summation theorem.

OR

2 Prove $\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k, n) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} A(k, n-k)$.

UNIT - II

3 State and prove the integral representation of ${}_pF_q$.

OR

3 State and prove the Saalschutz' theorem.

UNIT - III

4 Prove $ZJ_n'(z) = ZJ_{n-1}(z) - J_n(z)$.

OR

4 Prove $2n J_n(z) = Z[J_{n-1}(z) + J_{n+1}(z)]$.

https://pyqlelo.com

UNIT - IV

5 Establish ${}_1F_1(a;b;z) = e^z\ {}_1F_1(b-a;b;-z)$.

OR

5 Establish

$$(a-b+1)\ {}_1F_1(a;b;z)$$
$$= a_1F_1(a+1;b;z) - (b-1)\ {}_1F_1(a;b-1;z)$$

UNIT - V

6 If $e^t\Psi(x,t) = \sum_{n=0}^{\infty} \sigma_n(x)t^n$ then for $n \geq 1$ prove that

$$x\sigma_n'(x) - n\sigma_n(x) = -\sigma_{n-1}(x),\ \sigma_0'(x) = 0.$$

OR

6 Derive the differential recurrence relation for generating function of the form $G(2xt - t^2)$.

SECTION - C

7 If $|z| < 1$ and $|z/(1-z)| < 1$ then prove that

$$F(a, b; c; z)$$
$$= (1-z)^{c-a-b}\ F(c-a, c-b; c; z)$$

8 If $\mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0$ and k is a positive integer then prove that

$$\int_0^t x^{\alpha-1}(t-x)^{\beta-1}\ {}_pF_q\left[\begin{matrix}(ap).\\(bq).\end{matrix} cx^k\right]dx$$
$$= B(\alpha,\beta)\,t^{\alpha+\beta-1}\ {}_{p+k}F_{q+k}\left[\begin{matrix}(ap).\ \frac{\alpha}{k}, \ldots, \frac{\alpha+k-1}{k}.\\(bq).\ \frac{\alpha+\beta}{k}, \ldots, \frac{\alpha+\beta+k-1}{k}.\end{matrix} ct^k\right]$$

9 Derive Bessel's differential equation.

10 Prove that

$$e^{-z}\ {}_1F_1[a; 2a; 2z] = {}_0F_1\left[-; a+\frac{1}{2}; \frac{z^2}{4}\right].$$

https://pyqlelo.com

11 If $\Psi(u) = \sum_{n=}^{\infty}$

$(1-t)^{-c}\ \Psi\left(\left(-\right.\right.$

prove that

https://www.uokonline.com