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M.A./M.Sc. (Sem. II)

Examination, 2018

MATHEMATICS

Paper Math-2-C(IV)

(Special Functions)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 28

This question paper will be divided into ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This section will carry 10 marks with 1 compulsory question comprising 10 short answer type questions (maximum 20 words answer) taking two questions from each unit. Each question shall be of one mark.

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Section-B ***Max. Marks-25***

This section will carry **10** *questions,* **2** *questions from each unit (answer about in* **250** *words). Students are instructed to attempt* ***five*** *questions by selecting* ***one*** *question from each unit.*

Section-C ***Max. Marks-35***

This section will carry **35** *marks with* ***five*** *long answer type questions comprising* ***one*** *compulsory question (question no.* **7***) of* **15** *marks and* ***four*** *questions of* **10** *marks each. Students are instructed to attempt total* ***three*** *questions with* ***one*** *compulsory question (answer about in* **500** *words) and any* ***two*** *more questions (answer about in* **400** *words) out of remaining four questions.*

Section A

1. (i) Give notation to rising sequential product :

$$a(a+1)(a+2) \ldots\ldots\ldots (a+n-1),\ n \geq 1.$$

(ii) Write the Gauss' hypergeometric differential equation.

(iii) Write the absolutely convergence conditions for ${}_pF_q$.

(iv) Write any general contiguous function relation for ${}_pF_q$.

(v) Define Bessel function by ${}_0F_1$.

(vi) Find out :

$$\frac{d}{dx}[z^{-n}J_n(z)].$$

(vii) State the Kummer's first formula.

(viii) If $Re(b) > Re(a) > 0$, write the integral form of ${}_1F_1(a, b, z)$.

(ix) Define generating function concept.

(x) If

$$e^{t}\psi(xt) = \sum_{n=0}^{\infty} \sigma_n(x)t^n, \ \sigma_0(x) = 0$$

then for $n \geq 1$, write the recurrence relation to all set $\sigma_n(x)$.

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Section B

UNIT-I

2. Establish the following contiguous function relation :

$$[a + (b + c)z]\,F = a(1 - z)\,F(a+) - C^{-1}(c - a)$$
$$(c - b)z\,F(c+).$$

Or

Prove that :

$${}_2F_1\left[\begin{matrix}-n/2, & (1-n)/2; \\ b + \frac{1}{2} & ;\end{matrix}\ 1\right] = \frac{2^n(b)_n}{(2b)_n}.$$

UNIT-II

3. If n is a non-negative integer and if a, b are independent of n, then prove that :

$${}_3F_2\left[\begin{matrix}-n, a + n, \frac{1}{2} + \frac{q}{2} - b; \\ 1 + a - b, \frac{a}{2} + \frac{1}{2};\end{matrix}\ 1\right] = \frac{(b)_n}{(1 + a - b)_n}.$$

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Or

If $p \leq q + 1$, $\text{Re}(b_1) > \text{Re}(a_1) > 0$; $b_1, b_2, \ldots\ldots$, $b_j \neq 0, -1, -2 \ldots\ldots$ and if $|z| < 1$ then prove that :

$$ {}_pF_q\left[\begin{matrix} a_1, \ldots\ldots, a_p; \\ b_1, \ldots\ldots, b_q; \end{matrix} z\right] $$

$$ = \frac{\Gamma(b_1)}{\Gamma(a_1)\,\Gamma(b_1 - a_1)} \int_0^1 t^{a_1-1}\,(1-t)^{b_1-a_1-1} \; {}_{p-1}F_{q-1}\left[\begin{matrix} a_2, \ldots\ldots, a_p; \\ b_2, \ldots\ldots, b_q; \end{matrix} zt\right] dt $$

UNIT-III

4. Prove that :

$$ 2n\, J_n(z) = z[J_{n-1}(z) + J_{n+1}(z)]. $$

Or

Prove for $z \neq 0$ and for all finit x :

$$ e^{\frac{1}{2}x\left(z-\frac{1}{2}\right)} = \sum_{n=-\infty}^{\infty} z^n J_n(x). $$

UNIT-IV

5. Show that :

$$ {}_1F_1(a; b; z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-t} t^{a-1}\; {}_0F_1(-; b; zt)dt. $$

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Or

Prove that :

$$ {}_1F_1(a; b; z) = e^z\, {}_1F_1(b-a; b; -z). $$

UNIT-V

6. If

$$ e^t \psi(xt) = \sum_{n=0}^{\infty} \sigma_n(x) t^n, \quad \psi(u) = \sum_{n=0}^{\infty} r_n u^n, $$

then for arbitrary C, prove that :

$$ (1-t)^{-c} F\left(\frac{xt}{1-t}\right) = \sum_{n=0}^{\infty} (c)_n \sigma_n(x) t^n $$

where http://www.uokonline.com

$$ F(u) = \sum_{n=0}^{\infty} (c)_n r_n u^n. $$

Or

Let

$$ \psi(u) = \sum_{n=0}^{\infty} r_n u^n, \; r_0 \neq 0 $$

and the polynomial $f_n(x)$ by

$$ (1-t)^{-c} \psi\left(\frac{-4xt}{(1-t)^2}\right) = \sum_{n=0}^{\infty} f_n(x) t^n $$

then prove that :

$$ f_n(x) = \frac{(c)_n}{n!} \sum_{k=0}^{n} \frac{(-n)_k\,(c+n)_k\, r_k x^k}{\left(\frac{c}{2}\right)_k \left(\frac{c}{2}+\frac{1}{2}\right)_k}. $$

Section C

7. (a) Prove that : 10

$$(1-z)^{-1}\,{}_2F_1\left[\begin{matrix}a, c-b; \\ c;\end{matrix}\ \frac{-z}{1-z}\right] = {}_2F_1[a; b; c; z]$$

for $|z| < 1$ and $\left|\frac{z}{1-z}\right| < 1$.

(b) State and prove the Vander Mount's theorem. 5

8. If n is a non-negative integer and if b and c are independent of n, then prove that :

$${}_3F_2\left[\begin{matrix}-n, b, c; \\ 1-b-n, 1-c-n;\end{matrix}\ x\right] = (1-x)^n\,{}_3F_2\left[\begin{matrix}\frac{-n}{2}, \frac{-n}{2}+\frac{1}{2}, 1-b-c-n; \\ 1-b-n, 1-c-n;\end{matrix}\ \frac{-4x}{(1-x)^2}\right]$$

9. For integral n, prove that :

$$J_n(z) = \frac{1}{\pi}\int_0^{\pi}\cos(n\theta - z\sin\theta)d\theta.$$

10. If $2a$ is not an odd integer less than zero, then prove that :

$$e^{-z}\,{}_1F_1(a; 2a; 2z) = {}_0F_1\left(-; a+\frac{1}{2}; \frac{z^2}{4}\right).$$

11. From

$$A(t)\exp\left(\frac{-xt}{1-t}\right) = \sum_{n=0}^{\infty} y_n(x)t^n$$

it follows that $y_0'(x) = 0$, then prove for $n \geq 1$,

$$y_n'(x) = y_{n-1}'(x) - y_{n-1}(x)$$

and

$$y_n'(x) = -\sum_{k=0}^{n-1} y_k(x).$$

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