KK-5711

M.A./M.Sc. (Sem.-III) Examination, December - 2024

MATHEMATICS

Paper - MATH - 3C1

Analysis – III

Time Allowed : **Three Hours**

Maximum Marks : **100**

Note : This question paper contains **two** sections as under :

Section-A **Max. Marks-20**

This section will have **10** *questions, having* **02** *questions from each unit with no internal choice. Answer in about* **20** *words. All questions will have equal marks.* **(10×02=20)**

Section-B **Max. Marks-80**

This section will have **10** *questions, having* **02** *questions from each unit, one question to be attempted from each unit. Words limit approximately* **400-500** *words. Questions may have sub divisions. All questions will have equal marks.* **(16×05=80)**

SECTION – A

1 (i) Define bounded metric space.

(ii) State Cantor's intersection theorem.

(iii) Define Banach space and dense set.

(iv) Define Quotient space in Banach space.

(v) Define convex set and Hilbert space.

(vi) Define continuous linear transformation.

(vii) Define orthogonal complement.

(viii) State Bessel's inequality.

(ix) Define conjugate space.

(x) Define adjoint of an operator

SECTION - B

UNIT - I

2 (a) If (X, d) be a metric space and d^* be defined as

$$d^*(x,y) = \frac{Md(x,y)}{1+d(x,y)}; x, y \in X$$

and $M > 0$ then (X, d^*) is also a metric space, prove.

(b) Prove that in a metric space each closed sphere is a closed set.

OR

3 Define the term set of first category and then state and prove Baire's category theorem.

UNIT - II

4 (a) Let $x=(x_1,\ldots,x_n)$ and $y=(y_1,\ldots,y_n)$ be n-tuples of scalars (real and complex) and norm is defined as

$$\| x \| = \left[\sum_{i=1}^{n} | x_i |^2 \right]^{1/2}$$, then show that

$$\sum_{i=1}^{n} | x_i y_i | \leq \| x \| . \| y \| .$$

(b) Let N be a non zero normed linear space. then show that N is a Banach space $\Leftrightarrow$ $\{x \in N : \| x \| \leq 1\}$ is complete.

OR

5 Let M be a closed linear subspace of a normed linear space N and let $x_0 \notin M$. If $d = d(x_0, M)$ then there exists a functional

$f_0 \in N^*$ such that $f_0(M) = 0$. $f_0(x_0) = 1$.

$$\| f_0 \| = \frac{1}{d}$$

UNIT - III

6 Show that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

OR

7 Let B be a complex Banach space in which norm obeys the parallelogram law and an inner product in B is defined by

$$4(x,y) = \| x+y \|^2 - \| x-y \|^2 + i \| x+iy \|^2 - i \| x-iy \|^2,$$

then B is a Hilbert space.

UNIT - IV

8 State and prove Gram-Schmidth orthogonalization process theorem.

OR

9 Let $\{e_i\}$ be an orthonormal set in a Hilbert space H and x be any vector in H, then prove that $x - \sum_i (x, e_i) e_i \perp e_j \; \forall \; j$.

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UNIT - V

10 Let H be a Hilbert space and f be an arbitrary functional in H^*, then there exists a unique vector y in H such that

$$f = f_y \text{ i.e. } f(x) = (x, y) \ \forall x \in H .$$

OR

11 (a) An arbitrary operator T on a Hilbert space H can be uniquely expressed as $T = T_1 + iT_2$, where T_1 and T_2 are self adjoint operators in H.

(b) If P is the projection on the closed linear subspace M of H, then prove that

$$x \in M \Leftrightarrow P_x = x \Leftrightarrow \| P_x \| = \| x \| .$$