KR-5713

M.A./M.Sc. (Sem.-III) Examination, December - 2024

MATHEMATICS

Paper - MATH - 3O2

Fluid Dynamics-I

Time Allowed : **Three Hours**

Maximum Marks : **100**

Note : *This question paper contains* ***two*** *sections as under :*

https://pyqlelo.com

Section-A **Max. Marks-20**

This section will have ***10*** *questions, having* ***02*** *questions from each unit with no internal choice. Answer in about* ***20*** *words. All questions will have equal marks.* ***(10×02=20)***

Section-B **Max. Marks-80**

This section will have ***10*** *questions, having* ***02*** *questions from each unit, one question to be attempted from each unit. Words limit approximately* ***400-500*** *words. Questions may have sub divisions. All questions will have equal marks.* ***(16×05=80)***

https://pyqello.com

SECTION – A

1 (i) Define stream lines.

(ii) Define vorticity of flow.

(iii) Fluid motion is described by two methods, namely, the Lagrangian method and the Eulerian method, state the difference between the two method.

(iv) Bernoulli's equation is applicable between any two points in a flow field when the flow is _____.

(v) Define Lagrange's stream function or current junction.

(vi) Explain the physical meaning of the value of stream function at a point.

(vii) Define Doublets in two dimension.

(viii) State Milne-Thomson circle theorems.

(ix) Define two dimensional steady motion.

(x) Define an ideal fluid.

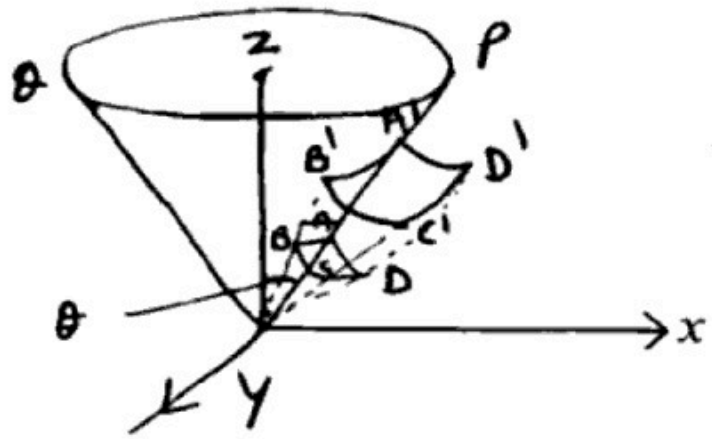
2×10=20

SECTION - B

UNIT - I

2 (a) If the lines of motion are curves of cones having their vertices at the origin and axis of z for common axis. Prove that the equation of continuity is

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial r}(pq_r) + \frac{2pq_r}{r} + \frac{\operatorname{cosec}\theta}{r}\frac{\partial}{\partial \phi}(pq_\phi) = 0$$



8

(b) Find the stream lines and path of the particles of the two dimensional velocity field

$$u = \frac{x}{1+t},\ v = y,\ w = o$$

8

OR

(a) Show that $\frac{x^2}{a^2} f(t) + \frac{y^2}{b^2} \frac{1}{f(t)} = 1$ is a possible form of boundary surface of a liquid.

8

(b) Show that

$$u = \frac{-2xyz}{(x^2+y^2)^2},\ v = \frac{(x^2-y^2)z}{(x^2+y^2)^2},\ w = \frac{y}{x^2+y^2}$$

are the velocity components of a possible liquid motion. Is this motion irrotational?

8

UNIT - II

3 (a) Consider a closed surface s such that it encloses a non-viscous fluids and is in motion with the fluid. Derive Euler's dynamical equations of motions in Cartesian coordinates. **8**

(b) Air obeying Boyle's law is in motion in a uniform tube of small section, prove that if ρ be the density and v the velocity at a distance x from a fixed point at time t is

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left[(v^2 + k)\rho\right].$$

8

OR

(a) State and prove Bernoullis theorem. **8**

(b) Steam is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d: if V and v be the corresponding velocities of the steam and if the motion be supposed to be that of divergence from the vertex of the cone, prove that

$$\frac{v}{V} = \frac{D^2}{d^2} e^{(v^2 - V^2)/2k}$$

where k is the pressure divided by the density and supposed constant. **8**

UNIT - III

4 (a) A mass of liquid surrounds solid sphere of radius a and its outer surface which is a concentric sphere of radius b is subject to a given constant pressure π, no other force being in action on the liquid. The solid sphere suddenly shrinks into a concentric sphere. Determine the subsequent motion. **8**

(b) In an irrotational motion in two dimensions, prove that

$$\left(\frac{\partial q}{\partial x}\right)^2 + \left(\frac{\partial q}{\partial y}\right)^2 = q\overline{\nabla}^2 q.$$

8

OR

(a) Find the condition that a function $f(x, y, \lambda) = 0$ is a possible system of stream lines for steady irrotational fluid motion when λ is a variable parameter. **8**

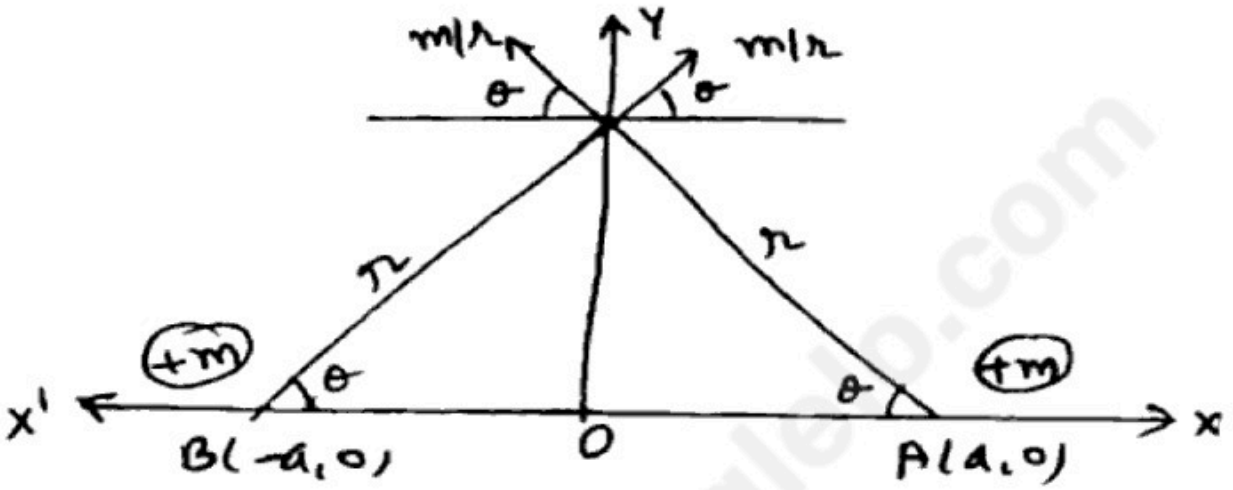
(b) Show that the velocity potential

$$\phi = \frac{1}{2}\log\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}$$

gives a possible motion and determine the stream lines. **8**

UNIT - IV

5 (a) Show that the image of a source with respect to a straight line is an equal source placed at equal distance on the other side of the line.



8

(b) What arrangements of sources and sinks will give rise to the function

$$w = \log\left(z - \frac{a^2}{z}\right).$$

Draw a rough sketch of stream lines. Prove that two stream lines sub-divide into the circle $r = a$ and the axis of y.

8

OR

(a) Find the image of a Doublet with respect to a straight line.

8

https://www.uokonline.com

(b) Two sources each of strength m are placed at the points $(-a, 0)$, (a, o) and a sink of strength $-2m$ at the origin. Show that the stream lines are the curves

$$(x^2 + y^2) = a^2(x^2 - y^2 + \lambda xy)$$

where λ is a variable parameter. Show also that the fluid speed at any point is $\frac{2ma^2}{r_1 r_2 r_3}$, where r_1, r_2, r_3 are the distances of the point from the sources and sink.

8

UNIT - V

6 State any prove Blosius theorem in a steady two dimensional motion.

16

OR

A circular cylinder is moving along x-axis in liquid at rest at infinity. Calculate the forces on the cylinder owing to the pressure of the liquid.

16