UC-5719

M. A. / M. Sc. (Sem.-IV)
Examination, June-2023

MATHEMATICS

PAPER : MATH 4 C(vi)

Functional Analysis - II

Time Allowed : Three Hours

Maximum Marks : 70
Min. Pass Marks : 28

Note : This question paper contains three sections as under :

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Section-A **Max. Marks-10**

This section contains 01 compulsory question comprising 10 short answer type questions (maximum 20 words answer) taking two questions from each unit. Each question shall be of one mark.

Section-B **Max. Marks-25**

This section contains 10 questions, 02 questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question from each unit. Each question shall be of five marks.

Section-C **Max. Marks-35**

This section contains five long answer type questions comprising one compulsory question (question no.12) of 15 marks and four questions of 10 marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in 500 words) and any two more questions (answer about in 400 words) out of remaining four questions.

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SECTION - A

1 (i) Define bounded linear transformation.

(ii) Define second conjugate space of normed linear transformation.

(iii) Define closed linear transformation.

(iv) Define a projection on a Banach space.

(v) Define inner product space.

(vi) Define parallelogram law.

(vii) Define orthogonal set.

(viii) State Bessel's inequality.

(ix) Define unitary operator.

(x) Define invariance.

SECTION - B

UNIT - I

2 Let N and N' be normed linear spaces and $T : N \to N'$ be a linear transformation. Then show that T is continuous either at each point of N or at no point of N.

3 Let N and N' be normed linear spaces and $T : N \to N'$ be a linear transformation. Then show that Kernal of T $\text{Ker}(T)$ is a subspace of N and $\text{Ker}(T)$ is closed if T is continuous.

UNIT - II

4 State and prove open mapping theorem.

5 Let N and N' be normed linear spaces and M be a subspace of N. Then show that a linear transformation $T : M \to N'$ is closed if and only if its graph T_G is closed.

UNIT - III

6 Show that every inner product space is a normed linear space.

https://pyqlelo.com

7 If x and y are any two vectors in a Hilbert space H, then show that $|(x, y)| \leq \|x\| \|y\|$.

UNIT - IV

8 Let S be a non empty subset of a Hilbert space H, then show that $S^{\perp}$ is a closed linear subspace of H. https://www.uokonline.com

9 Let M and N be closed linear subspace of a Hilbert space H such that $M \perp N$. Then show that the linear subspace $M+N$ is also closed.

UNIT - V

10 Show that an operator T on a Hilbert space H is self adjoint if and only if (Tx, x) is real for all x.

11 If M and N are closed linear subspaces of a Hilbert space H and P and Q are projections of M and N respectively, then show that $M \perp N \Leftrightarrow PQ = O \Leftrightarrow QP = O$.

SECTION - C

12 Let H be a Hilbert space and f be an arbitrary functional in H^*. Then show that there exists a unique vector y in H such that $f(x) = (x, y) \forall x \in H$.

13 State and prove Hahn Banach theorem.

14 Let B be a Banach space and ~~N be a normed~~ linear space. If $\{T_i\}$ is a non empty set of continuous linear transformation of B into N with the property that $\{T_i(x)\}$ is a bounded subset of N for each x in B. Then show that $\{\|T_i\|\}$ is a bounded set of real numbers.

15 Let M be closed linear subspace of a Hilbert space. Let x be a vector not in M and d be the distance of x from M. Then show that there exists a unique vector y_o in M such that $\|x - y_o\| = d$.

16 If P is the projection on the closed linear subspace M of H. Then show that $x \in M \Leftrightarrow Px = x \Leftrightarrow \|Px\| = \|x\|$.

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