UC-5721

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M. A. / M. Sc. (Sem. IV)
Examination, June - 2023

MATHEMATICS

Paper - MATH 4 O(i)

~~Operations Research - II~~

Time Allowed : **Three Hours**

Maximum Marks : **70**
Min. Pass Marks : **28**

Note : *Non-programmable Scientific Calculator is allowed in this paper.*

Note : *This question paper is divided into* ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This section contains ***01*** *compulsory question comprising* ***10*** *short answer type questions (maximum* ***20*** *words answer) taking* ***two*** *questions from each unit. Each question shall be of* ***one*** *mark.*

Section-B ***Max. Marks-25***

This section contain ***10*** *questions,* ***02*** *questions from each unit (answer about in* ***250*** *words). Students are instructed to attempt* ***five*** *questions by selecting* ***one*** *question from each unit. Each questions shall be of five marks.*

Section-C ***Max. Marks-35***

This section cotains ***five*** *long answer type questions comprising* ***one compulsory*** *question (question no.* ***07)*** *of* ***15*** *marks and four questions of* ***10*** *marks each. Students are instructed to attempt total* ***three*** *questions with* ***one compulsory*** *question (answer about in* ***500*** *words) and any* ***two*** *more questions (answer about in* ***400*** *words) out of remaining four questions.*

https://pyqlive.com

SECTION - A

1 (i) State Bellman's principle of optimality.
(ii) What is Dynamic programming?
(iii) Define lead time.
(iv) What is inventory?
(v) Define non linear programming.
(vi) Define absolute maximum value in the absence of constraints.
(vii) Define separable convex programming problem.
(viii) Define Kuhn-Tucker necessary condition for an optimal solution to a quadratic programming problem.
(ix) Define three types of customer behaviour in queueing theory.
(x) Define mean arrival rate.

SECTION - B

UNIT - I

2 Use dynamic programming to show that $\sum_{i=1}^{n} p_i \log p_i$ subject to $\sum_{i=1}^{n} p_i = 1, p_i \geq 0$ is minimum, when $p_1 = p_2 = \ldots\ldots = p_n = \frac{1}{n}$

OR

Find the value of : $\max(y_1\ y_2\ y_3)$

subject to $y_1 + y_2 + y_3 = 5$

and. $y_1, y_2, y_3 \geq 0$

UNIT - II

3 A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that, when he starts a production run, he can produce 25,000 bearing per day. The cost of holding a bearing in stock for one year is 2 paisa, and the set-up cost of a production run is ₹ 18. How frequently should production run be made?

OR

An engineering factory consumes 50,000 units of a component per year. The ordering, receiving and handling cost are ₹ 3 per order while the truck cost are ₹ 12 per order. Other details are as follows :
Interest cost = ₹ 0.06 per unit per year, Deterioration and observation cost = ₹ 0.004 per unit per year, Storage cost = ₹ 1,000 per year for 50,000 units. Calculate the Economic Order Quantity.

UNIT - III

4 Write the Kuhn-Tucker conditions for the problem :

Max. $Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

s.t. $x_1 + x_2 \le 2$

$2x_1 + 3x_2 \le 12$

and $x_1, x_2 \ge 0$, hence solve it.

OR

Use Lagrangian multiplier method to solve the NLPP :

Min. $Z = 2x_1^2 + 2x_2^2 + 2x_3^2 - 24x_1 - 8x_2 - 12x_3 + 10$

S. t. $x_1 + x_2 + x_3 = 11$

$x_1, x_2, x_3 \ge 0$

UNIT - IV

5 Solve the following QPP by Wolfe's method

Max. $Z = 2x_1 + x_2 - x_1^2$

S.t. $2x_1 + 3x_2 \le 6$

$2x_1 + x_2 \le 4$

and $x_1 \ge 0, x_2 \ge 0$

OR

Solve the following QPP by Beale's method :

Max. $f(x) = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

S.t. $x_1 + 2x_2 \le 10$

$x_1 + x_2 \le 9$

and $x_1, x_2 \ge 0$

UNIT - V

6 If for a period of 2 hours in the day (7 AM to 9 AM), customers arrive in a barber's shop that has space to accomodate only 4 customers. Arrival rate of customers is 3m per hour and *service time* is 36 minutes/customer for the above period, find :

(a) The probability that there is no customers in the shop.

(b) Average number of customers in the shop.

OR

The time spent by a repairman on his job has an exponential distribution with mean 30 minutes. If he repairs set in the order in which they come in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

SECTION - C

7 Deduce the difference equations for birth and death model. Obtain steady state solution.

8 Use dynamic programming to solve :

$$\text{Min. } Z = \sum_{i=1}^{n} x_i$$

$$\text{S.t. } \sum_{i=1}^{n} x_i = b \text{ and } x_i \geq 0, i = 1, 2, \ldots, n$$

Hence or otherwise minimise

$x_1^2 + x_2^2 + x_3^2$

s.t. $x_1 + x_2 + x_3 \geq 15$

and $x_1, x_2, x_3 \geq 0$

9 Five jobs must go through three Machines M_1, M_2 and M_3 in the order M_1, M_2, M_3. Find the sequence that will minimize the total elapsed time :

Jobs → / Machine ↓	1	2	3	4	5
M_1	5	7	6	9	5
M_2	2	1	4	5	3
M_3	3	7	5	6	7

10 Find approximate optimal solution of the separable programming problem :

$2x_1^2 + 3x_2^2 \leq 6, x_1, x_2 \geq 0$

Max. $Z = 2x_1 - x_1^2 + x_2$

11 A contractor of second hand motor truck used to maintain a stock of trucks every month. The demand of the trucks is at a relatively constant rate but not in a constant size. The demand follows the following probability distribution. The holding cost of an old truck in stock for one month is Rs. 100 and the *penalty* for a truck if not supplied on the demand is Rs. 1,000. Determine to optimal size of stock for the contractor :

Demand (r)	Probability $P(r)$
0	0.40
1	0.24
2	0.20
3	0.10
4	0.05
5	0.01
6	0.00

or more.