UC-5720

M. A. / M. Sc. (Sem. IV)
Examination, June - 2023

MATHEMATICS

PAPER - MATH 4 C (VII)

TOPOLOGY - II

Time Allowed : **Three Hours**

Maximum Marks : **70**
Min. Pass Marks : **28**

Note : *This question paper will be divided into three sections as under :*

Section-A

Max. Marks - 10

This section contains 01 compulsory question comprising 10 short answer type questions (maximum 20 words answer) taking two questions from each unit. Each question shall be of one mark.

Section-B

Max. Marks-25

This section contains 10 questions, 02 questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question from each unit. Each question shall be of five marks.

Section-C

Max. Marks-35

This section contains five long answer type questions comprising one compulsory question (question no. 07) of 15 marks and four questions of 10 marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in 500 words) and any two more questions (answer about in 400 words) out of remaining four questions.

SECTION - A

1 (i) Define Totally bounded metric space.

(ii) Define Lebesgue numbers for covers.

(iii) Explain locally connected space.

(iv) On the real line, consider the sets $A = (0, 1)$, $B = (1, 2)$ and $C = [1, 2)$. Show that A and B are separated but A and C are not.

(v) Define Tychonoff topology.

(vi) Define Diagonal map.

(vii) Define Cofinal subset with an example.

(viii) Define Ultranet.

(ix) Show that every filter has the finite intersection property.

(x) Define base of a filter.

SECTION - B

UNIT - I

2 Show that every compact subset of a metric space is bounded.

OR

Show that every sequentially compact metric space has the Bolzano Weierstrass Property.

UNIT - II

3 Show that union of any family of connected sets having a non-empty intersection is connected.

OR

Show that components of a totally disconnected space are the singleton sets.

https://www.uokonline.com

UNIT - III

4 Prove that product space of two first countable topological spaces is first countable.

OR

Let X and Y be two topological spaces. Then show that the product space $X \times Y$ is connected iff X and Y are connected.

UNIT - IV

5 Let $(X, \Im)$ be a topological space and let $A \subset X$.

Then prove that a point $x \in X$ is an adherent point of A iff there exists a net in A which converges to x.

OR

If every net in a topological space $(X, \Im)$ has a cluster point then show that $(X, \Im)$ is compact.

UNIT - V

6 Let $(X, \Im)$ be a topological space. Then prove that a filter base $\mathcal{B}$ on X converges to a point $x \in X$ iff every *nbd* of x contains a member of $\mathcal{B}$.

OR

Show that every filter $\mathcal{G}$ on X is the intersection of all the ultra filters finer than $\mathcal{G}$.

SECTION - C

7 Show that every open covering of sequentially compact metric space has a Lebesgue number.

8 Show that a topological space $(X, \Im)$ is locally connected iff the components of every open subspace of X are open in X.

https://pyqlearn.com

9 Prove that the product space $\Pi\{X_\alpha : \alpha \in \Lambda\}$ is Hausdorff iff each co-ordinate space X_α is Hausdorff.

10 Show that a filter $\mathcal{F}$ on a non-empty set X is an ultra filter on X iff $\mathcal{F}$ contains all those subsets of X which intersects every member of $\mathcal{F}$.

11 Show that a topological space $(X, \mathfrak{I})$ is a Hausdorff space iff every net in X can converge to at most one point.

https://pyqlelo.com

https://www.uokonline.com