U-5719

M. A. / M. Sc. (Sem. - IV) Examination, June-2025

MATHEMATICS

Paper - MATH 4C1

Topology

Time Allowed : **Three Hours**

Max. Marks : **100**

*This question paper contains **two** sections as under :*

Section-A ***Max. Marks-20***

This section will have 10 questions, having 02 questions from each unit with no internal choice. Answer in about 20 words. All questions will have equal marks. (02×10=20)

Section-B ***Max. Marks-80***

This section will have 10 questions, having 02 questions from each unit, one question to be attempted from each unit. Word limit approximately 400-500 words. Questions may have sub divisions. All questions will have equal marks. (16×05=80)

SECTION - A

1 (i) Define totally bounded metric space.
(ii) Define derived sets.
(iii) Explain locally connected space.
(iv) Define sequential compactness.
(v) Define Tychonoff topology.
(vi) Define Hausdorff spaces.
(vii) Define residual subset.
(viii) Define cofinal subset with an example.
(ix) Explain product topology.
(x) Define separated space.

SECTION - B

UNIT - I

2 (a) A subset of a topological space is open iff it is a nbd of each of its points.

(b) P.T. the intersection of an arbitrary family of topologies a set X is a topology for X.

OR

(a) Let (X, T) be a topology space and let $A \subset X$. Prove that $F_x(A) = \phi$ iff A is open as well as closed.

(b) Show that $A \cup D(A)$ is closed set. Also show that $A \cup D(A)$ is smallest closed subset of X containing A.

UNIT - II

3 (a) Prove that the function $f:(X,T)\to(Y,U)$ is continuous iff $f^{-1}(V)$ is open in X for each open set V in Y.

(b) Let f be a mapping of (X, T) into (Y, V). Then f is continuous iff

$$\left[f^{-1}(A)\right]^{\circ} \supset f^{-1}\left[A^{\circ}\right], \forall\ A \subset Y$$

OR

(a) Let (X, T) be a topological space and Y be a subset of X. Then the collection $T_y = \{G \cap Y : G \in J\}$ is a topology for Y.

(b) Let (X, T) be a topological space. A sub collection B of T is a base for T iff every T-open set can be expressed as the union of members of B. Prove it.

UNIT - III

4 (a) Let X and Y be two topological spaces. Then show that the product space $X \times Y$ is connected.

(b) Show that every metric space is T_2 space.

OR

A metric space is separable iff it is second countable. Prove.

UNIT - IV

5 (a) The product space of two second countable space is second countable.

(b) A metric space X is compact iff it is sequentially compact. Prove.

OR

Prove that the product space $\pi\{X_\alpha : \alpha \in \Lambda\}$ is Hausforff iff each co-ordinate space X_α is Hausdorff.

UNIT - V

6 (a) Show that a topological space (X, J) is locally connected iff the components of every open subspace of X are open in X.

(b) A topological space X is disconnected iff there exists a non empty proper subset of X which is both open and closed in X. Prove.

OR

(a) Prove that a subset of R is connected if and only if its is an interval.

(b) Prove that the finite product of locally connected space is locally connected.

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