This question paper contains 8 printed pages]

1651

M.A./M.Sc. (Previous) Examination, 2018

MATHEMATICS

Paper I

(Advanced Algebra)

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains three sections as under :

Section-A. **खण्ड 'अ'** ***Max. Marks-10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*

इस खण्ड में एक अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से 2 लघु प्रश्न लेते हुए कुल 10 लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर 20 शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान

*This section contains **10** questions having **2** questions from each unit. Answer **5** questions (**250** words each) selecting **one** question from each unit. **All** questions carry equal marks.*

*इस खण्ड में प्रत्येक इकाई से 2 प्रश्न लेते हुए कुल 10 प्रश्न हैं। प्रत्येक इकाई से **एक** प्रश्न का चयन करते हुए, कुल **5** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो। **सभी** प्रश्नों के अंक समान हैं।*

Section-C **खण्ड 'स'** ***Max. Marks-40***

*This sect on contains **4** descriptive type questions (question ma) have sub-division) covering all units but not more than one question from each unit. Answer any **two** questions (**500** words each). **All** questions carry equal marks.*

*इस खण्ड में **4** वर्णनात्मक प्रश्न होंगे (प्रश्न के उप-भाग भी हो सकते हैं) जो सभी इकाइयों में से दिये जायेंगे, किन्तु एक इकाई से एक से अधिक प्रश्न नहीं होगा। कोई दो प्रश्नों के उत्तर दिये जाने हैं। प्रत्येक प्रश्न का उत्तर **500** शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान हैं।*

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Section A

1. (*i*) Define Sylow *p*-subgroup.

(*ii*) Write a class equation for a finite group.

(*iii*) Define a principal Ideal Ring.

(*iv*) Define an Algebraic Extension.

(*v*) Define a second Dual Space.

(*vi*) Define a non-singular transformation.

(*vii*) Define a similarity matrix.

(*viii*) Define an Index of nilpotency.

(*ix*) Define an Orthogonal Complements.

(*x*) Define a Hermitian form. 10×1=10

Section B

UNIT-I

2. If G is non-abelian group of order P^3, where P is prime, then show that order of centre of G is P. 10

3. Show that the symmetric group S_3 of degree 3 is solvable. 10

UNIT-II

4. Prove that an Ideal I of a commutative ring R with unity is prime if and only if R/I is an Integral Domain. 10

Or

5. Show that any field of characteristic 0 (zero) is perfect. 10

UNIT-III

6. Let $t : R^3 \to R^3$ be a linear transformation as

$t(x, y, z) = (x + 2y - z, y + z, z + y - 2z)$

Find the rank and Nullity of *t*. 10

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7. Let $\dim V(F) = n$ and $B = \{\overline{b_1}, \overline{b_2}, \ldots\ldots, \overline{b_n}\}$, $B^* = \{x_1, x_2, \ldots\ldots, x_n\}$ are the basis and dual basis of V respectively. Then prove that $x_1 b_1 + x_2 b_2 + \ldots\ldots + x_n b_n = I_V$. (Identity map from V to V'). 10

UNIT-IV

8. Express the following expression as a linear polynomial : 10

$$A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2, \; A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}.$$

Or

9. Show that the following matrix is diagonalizable : 10

$$A = \begin{bmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}.$$

UNIT-V

10. If the field F of characteristic $\neq 2$, then prove that every symmetric bilinear form on V(F) is uniquely determined by the corresponding quadratic form. 10

Or

11. Prove that an orthogonal set of non-zero vectors in an inner product space V(F) is linearly independent. 10

Section C

12. (a) Prove that every homomorphic image of a group G is isomorphic to some quotient

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prove that the number of elements conjugate to a in G is the index of the normalizer of a in G *i.e.* : 10

$$O(C|[a]) = \frac{O(G)}{O[N(a)]}.$$

13. (*a*) Prove that a commutative ring with unity is a field if it is simple. 10

(*b*) Prove that a non-zero element a in a Euclidean ring R is a unit if and only if $d(a) = d(1)$ where 1 is the unity element of R. 10

14. (*a*) Find the minimal polynomial for real matrix : 10

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}.$$

(*b*) Let t be a linear operator on V(F). Then prove that the following are t-invariant :

(*i*) Null space of t

(*ii*) Range space of t. 5+5=10

15. If $B = \{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n\}$ is any finite orthonormal set in an inner product space V(F) and if $\beta \in V(F)$, then prove that :

(*a*) $\sum_{i=1}^{n} |(\beta, \alpha_i)|^2 \leq \|\beta\|^2$

(*b*) $\sum_{i=1}^{n} |(\beta, \alpha_i)|^2 = \|\beta\|^2 \Leftrightarrow \beta = L(B)$

(*c*) Vector $\gamma = \beta - \sum a_i \alpha_i$ is orthogonal to each $\alpha_j \in B (1 \leq j \leq n)$. 5+5=10

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