This question paper contains 8 printed pages]

1651

M.A./M.Sc. (Previous) Examination, 2019

MATHEMATICS

Paper I

(Advanced Algebra)

Time Allowed : Three Hours

Maximum Marks : 100

*This question paper contains **three** sections as under :*

Section-A **खण्ड 'अ'** ***Max. Marks-10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*

*इस खण्ड में **एक** अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से **2** लघु प्रश्न लेते हुए कुल **10** लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर **20** शब्दों से अधिक न हो। **सभी** प्रश्नों के अंक समान हैं।*

Section-B **खण्ड 'ब'** ***Max. Marks-50***

*This section contains **10** questions having **2** questions from each unit. Answer **5** questions (**250** words each) selecting **one** question from each unit. **All** questions carry equal marks.*

*इस खण्ड में प्रत्येक इकाई से **2** प्रश्न लेते हुए कुल **10** प्रश्न हैं। प्रत्येक इकाई से **एक** प्रश्न का चयन करते हुए, कुल **5** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो। **सभी** प्रश्नों के अंक समान हैं।*

Section-C **खण्ड 'स'** ***Max. Marks-40***

*This section contains **4** descriptive type questions (questions may have sub-divisions) covering all units but not more than one question from each unit. Answer any **two** questions (**500** words each). **All** questions carry equal marks.*

*इस खण्ड में **4** वर्णनात्मक प्रश्न होंगे (प्रश्नों के उप-भाग भी हो सकते हैं) जो सभी इकाइयों में से दिये जायेंगे, किन्तु एक इकाई से एक से अधिक प्रश्न नहीं होगा। किन्हीं **दो** प्रश्नों के उत्तर दिये जाने हैं। प्रत्येक प्रश्न का उत्तर **500** शब्दों से अधिक न हो। **सभी** प्रश्नों के अंक समान हैं।*

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Section A

1. Define :

   (*i*) P-group

   (*ii*) Composition series

   (*iii*) Principal ideal

   (*iv*) Primitive polynomial

   (*v*) Singular transformation

   (*vi*) Projection

   (*vii*) Minimal polynomial

   (*viii*) Nilpotent operator

   (*ix*) Quadratic form

   (*x*) Inner product.

Section B

UNIT-I

2. Show that for any group G, $\frac{G}{\{e\}} \equiv G$.

3. Show that a group G is solvable iff $G^{(k)} = \{e\}$ for some $k \in N$.

UNIT-II

4. If S is an ideal of ring R with unity element, then show that S = R.

5. Show that an ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R iff a_0 is a prime element of R.

UNIT-III

6. Let $T : R^3 \to R^3$ be a linear transformation such that :

   $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.

   Find the rank and nullity of T.

7. Let

   $\{e_1 = (1, -1, 3), e_2 = (0, 1, -1), e_3 = (0, 3, -2)\}$

   be the basis of R^3. Find the dual basis $\{\phi_1, \phi_2, \phi_3\}$.

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UNIT-IV

8. Show that the minimal polynomial of a matrix A divides every polynomial that annihilates A.

9. Let $T : V \to V$ be a linear transformation. Let for $u \in V$, $T^k(u) = 0$ and $T^{k-1}(u) \neq 0$. Show that :

   (*i*) Set $S = \{u, T(u), \ldots\ldots, T^{k-1}(u)\}$ is linearly independent.

   (*ii*) Subspace W generated by S is T-invariant.

UNIT-V

10. Let f be a bilinear form on R^2 defined as $f((x_1, x_2), (y_1, y_2)) = 2x_1y_1 - 3x_1y_2 + x_2y_2$. Find the matrix of f in the basis $\{(1, 0), (1, 1)\}$.

11. Construct an orthonormal basis from the following basis of R^3 :

$\{(1, 0, 1), (1, 2, -2), (2, -1, 1)\}$.

Section C

12. (*i*) If for some prime P, $O(G) = P^2$, then show that G is abelian.

   (*ii*) Let R be a Euclidean ring. Then show that every element in R is either a unit in R or can be written as the product of finite number of prime elements in R.

13. (*i*) Given two polynomials $f(x)$ and $g(x) \neq 0$ in $F[x]$, show that there exist polynomials $q(x)$, $r(x)$ in $F[x]$ such that $f(x) = q(x)\, g(x) + r(x)$, where $r(x) = 0$ or $\deg r(x) < \deg g(x)$, F is a field.

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(*ii*) If L is a finite extension of K and if K is a finite extension of F, then show that L is a finite extension of F, where L, K, F are fields.

14. (*i*) Suppose vector space V has finite dimension and W is a subspace of V. Show that :

$$\dim W + \dim W^{\circ} = \dim V.$$

(*ii*) Diagonalise :

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

15. (*i*) If F is a field of characteristic $\neq 2$, then show that every symmetric bilinear form on V(F) can be uniquely determined by the corresponding quadratic form.

(*ii*) If α and β are any two vectors of real inner product space V, then show that :

$$\|\alpha\| = \|\beta\| \Rightarrow (\alpha + \beta) \perp (\alpha - \beta).$$

Give its geometrical interpretation also.

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