This question paper contains 7 printed pages]

1651

M.A/M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper I

(Advanced Algebra)

Time Allowed : Three Hours

Maximum Marks : 100

*This question paper contains **three** sections as under :*

Section-A ***Max. Marks-10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*

Section-B ***Max. Marks-50***

*This section contains **10** questions having **2** questions from each unit. Answer **5** questions (**250** words each) selecting one question from each unit. **All** questions carry equal marks.*

Section-C ***Max. Marks-40***

*This section contains **4** descriptive type questions (questions may have sub-divisions) covering all units but not more than one question from each unit. Answer any **2** questions (**500** words each). **All** questions carry equal marks.*

Section A

1. (*i*) Define the conjugate elements.

(*ii*) Define the solvable groups.

(*iii*) Define the principal ideal ring.

(*iv*) Define the Algebraic extension of a field.

(*v*) Define the adjoint of a linear transformation.

(*vi*) Define the bidual of a vector space.

(*vii*) Give the statement of Cayley-Hamilton theorem.

https://www.uokonline.com

(*viii*) Define the similar matrices.

(*ix*) Define the symmetric and skew-symmetric bilinear forms.

(*x*) Define the quadratic form.

Section B

UNIT-I

2. Let $f : G \to G'$ be a morphism of groups. Let N be a normal subgroup of G contained in the kernel of f, then there exists a unique morphism $\psi : \frac{G}{N} \to G'$ such that $f = \psi p$.

3. Let G be a finite group and p a prime number such that $p \mid o(G)\mid$. Then all Sylow p-subgroups of G are conjugate of each other.

UNIT-II

4. An ideal I of a commutative ring R, with unity is maximal iff the quotient Ring R/I is a field.

5. If F, K, L are three fields such that K is a finite extension of F and L is a finite extension of K, then L is a finite extension of F and [L : F] = [L : K] [K : F]

UNIT-III

6. If f be a linear transformation from a vector space V(F) onto V'(F), then

$$\frac{V}{k_{erf}} \cong I_m(f) \quad \text{or} \quad \frac{V}{k_{erf}} \cong V'.$$

7. Let W be a subspace of V(F), then :

$$\dim A(W) = \dim V - \dim W$$

UNIT-IV

8. Let V, V', V" be finite dimensional vector space over a field F and let B, B' and B" be their respective bases, then for linear transformation $t : V \to V'$ and $s : V' \to V''$.

$$M_{B''}^{B}(sot) = M_{B''}^{B'}(s)\, M_{B'}^{B}(t)$$

https://pyqlelo.com

9. Show that the eigen values of the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ are ± 1. Find the corresponding eigen vectors.

UNIT-V

10. If any linear transformation $X = BY$ is applied to a quadratic form q, then the resultant is another quadratic form q' whose matrix C can be represented as $C = B^T AB$.

11. Prove that $V^2(R)$ is an innerproduct space with an inner product defined on :

$$\alpha = (a_1, a_2),\ \beta = (b_1, b_2) \text{ by}$$

$$(\alpha, \beta) = a_1b_1 - a_2b_1 - a_1b_2 + 2a_2b_2$$

Section C

12. (a) If G is a finite group and $a \in G$, then the number of elements conjugate to "a" in G is the index of the normalizer of a in G *i.e.*,

$$O[a] = \frac{O(G)}{O[N(a)]}$$

(b) Any *two* composition series of a finite group G are equivalent *i.e.*, are of same length and have isomorphic factors.

13. (a) If $f(x)$ and $g(x) \neq 0$ are any two polynomials over a field F, then exist unique polynomials $q(x)$ and $r(x)$ in $F(x)$ such that :

$$f(x) = q(x)\ g(x) + r(x)$$

where either $r(x) = 0$ or deg $r(x)$ < deg $g(x)$.

(b) Let α, β be separable over K, then $K(\alpha, \beta)$ is a simple extension of k.

14. (a) If V(F) and V'(F) are n-dimensional vector spaces having their bases as $B = [b_1, b_2, \ldots, b_n]$ and $B' = [b_1', b_2', \ldots, b_n']$ respectively, then there exists a unique linear transformation $t : V \to V'$ such that :

$$t(b_i) = b_i', \qquad i = 1, 2, \ldots, n$$

https://pyqlelo.com

(*b*) Every square matrix satisfies its own characteristic equation :

$$|A - xI| = 0 \text{ or } \phi(A) = 0$$

i.e., $A^n + a_1A^{n-1} + a_2A^{n-2} + \ldots + a_nI = 0$

15. (*a*) Every finite dimensional inner product space has an orthonormal basis.

(*b*) If $\alpha, \beta \in V(F)$, then $|(\alpha, \beta)| \leq \|\alpha\|.\|\beta\|$

https://pyqlelo.com