U-1651

M. A. / M. Sc. (Prev.) Examination 2021

MATHEMATICS

Paper - I

(Advanced Algebra)

Time Allowed : **Three Hours**

Maximum Marks : **100**

Note : This questions paper contains three sections as under :

SECTION - A

1 Define :

(i) Class equation

(ii) Solvable group

(iii) Euclidean ring

(iv) Perfect field

(v) Range of Linear transformation

(vi) Annihilator

(vii) Eigen vector of linear transformation

(viii) Index of nilpotency

(ix) Skew symmetric bilinear form

(x) Orthonormal set

[1×10=10]

SECTION - B

UNIT - I

2 If H and K are subgroups of G with K normal in G, then show that $H \cap K$ is a normal subgroup of H and $\frac{HK}{K} \cong \frac{H}{(H \cap K)}$. [10]

https://pyqlele.com

3 Show that the two congugate classes $c[a]$ and $c[b]$ of a group G are either disjoint or identical. [10]

UNIT - II

4 Let R be a Eudidean ring and a non-zero, non-unit $a \in R$. Suppose that $a = s_1 s_2 \ldots s_m = s'_1 s'_2 \ldots s'_n$, where s_i and s'_j are prime elements of R. Then show that $m = n$ and each $s_i, 1 \le i \le m$ is an associate of some s'_j, $1 \le j \le n$ and conversely. [10]

5 If $\frac{K}{F}$ is a finite field extension, then show that it is an algebraic extension. [10]

UNIT - III

6 If $t : v \to v'$ is an isomorphism, then show that $\{v_1, v_2, \ldots, v_n\}$ is linearly independent if and only if $\{t(v_1), t(v_2), \ldots, t(v_n)\}$ is linearly independent. [10]

7 Let V be a finite dimensional vector space over a field F, W be a subspace and $A(W)$ is annihilator of W, then show that

$$\dim A(W) = \dim V - \dim W, \text{ and } W^* \cong \frac{V^*}{A(W)}$$

[10]

UNIT - IV

8 Let V be the vector space of all 2×2 matrices over F. Let $t : V \to V$ be linear transformation such that $t(X) = AX$, where $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Find matrix of t relative to the basis

$$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \text{ for } V.$$

[10]

9 If a square matrix A of order n, over a field F, has n distinct eigen values $\lambda_i, i = 1, 2, \ldots, n$. Show that there is an invertible matrix P such that

$$P^{-1} A P = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$$

[10]

https://www.uokonline.com

UNIT - V

10 Let V be a finite dimensional vector space over a subfield of the complex numbers, let f be a symmetric bilinear form on V. Then show that there is an ordered basis for V is which f is represented by a diagnonal matrix.

[10]

11 Let V be an inner product space. Then shows that $|<u,v>| \leq \|u\| \|v\|$ for $u, v \in V$. Further equality holds if and only if u and v are linearly dependent.

[10]

SECTION - C

12 (i) Show that any two subnormal series of a group G have subnormal refinements that are equivalents.

(ii) Show that every finite group G has a composition series.

[10+10=20]

13 (i) Let $t: V \to V'$ be a linear transformation and V is finite dimensional, then show that

$$\dim V = \text{rank}(t) + \text{nullity}(t)$$

(ii) It V is finite dimensional vector space, then show that there is a natural isomorphism of V onto V^{**}.

[10+10=20]

14 (i) Let V be an n-dimensional vector space over a field F with basis B and V' be an m-dimensional vector space with basis B'. Then show that

$$Hom(V, V') \cong M_{m \times n}(F),$$

where Hom (V, V') is space of linear transformations from V to V' and $M_{m \times n}(F)$ is the space of $m \times n$ matrices with coefficient in F.

(ii) Show that similar matrices have the same minimal polynomial.

[10+10=20]

5 (i) Show that every finite dimensional vector space V with an inner product has an orthonormal basis.

(ii) Show that a polynomial of degree n over a field F can have at most n roots in any extension field.

[10+10=20]

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