UB-1651

M. A. / M. Sc. (Previous) Examination, 2023

MATHEMATICS

Paper - I

ADVANCED ALGEBRA

Time Allowed : **Three Hours**

Maximum Marks : **100**

This question paper contains three sections as under :

Section-A खण्ड–अ **Max. Marks-10**

This section contains one compulsory question with 10 parts, having 2 parts from each unit, short answer in 20 words for each part. All questions carry equal marks.

इस खण्ड में एक अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से 02 लघु प्रश्न लेते हुये कुल 10 लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर 20 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान है।

Section-B खण्ड–ब **Max. Marks-50**

This section contains 10 questions having 2 questions from each unit. Answer 5 questions (250 words each) selecting one question from each unit. All questions carry equal marks.

इस खण्ड में प्रत्येक इकाई से 02 प्रश्न लेते हुये कुल 10 प्रश्न है। प्रत्येक इकाई से एक प्रश्न का चयन करते हुये कुल 5 प्रश्नो के उत्तर देने है। प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान है।

Section-C खण्ड–स **Max. Marks-40**

This section contain 4 descriptive type questions (questions may have sub division) covering all units but not more than one question from each unit. Answer any two questions. (500 words each). All questions carry equal marks.

इस खण्ड में 4 वर्णनात्मक प्रश्न होंगे। प्रश्न में भाग भी हो सकते हैं जो सभी इकाईयों मे से दिये जायेगें, किन्तु एक इकाई में से एक से अधिक प्रश्न नही होगा। कोई दो प्रश्नों के उत्तर दिये जाने है। प्रत्येक प्रश्न का उत्तर 500 शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान है।

SECTION - A

1 (i) Define Kernel of homomorphism.

(ii) Define Subnormal series.

(iii) Define Euclidean ring.

(iv) Define Simple extension.

(v) State rank-nullity theorem.

(vi) Define adjoint of a linear transformation.

(vii) Define similarity of linear transformation.

(viii) Define Nilpotent transformation.

(ix) Define Symmetric bilinear form.

(x) Define Orthogonal complements.

SECTION - B

UNIT - I

2 If G be a finite group and $a \in G$ then prove that

$$O(C[a]) = \frac{O(G)}{O[N(a)]} = [G : N(a)]$$

Where $C[a]$ be conjugate class of a and $N(a)$ be normalizer of a.

3 Prove that a group G is solvable if and only if $G^{(n)} = \{e\}$ for some $n \in N$, where e is identity element of group and $G^{(n)}$ is n^{th} derived subgroup of G.

UNIT - II

4 State and prove unique factorization theorem.

5 If K be a field extension of a field F then prove that an element $a \in K$ is algebraic over F if and only if $F(a)$ is finite extension of F.

UNIT - III

6 If F be the field of complex numbers and T be the function from F_3 to F_3 defined as $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2 - x_3, -x_1 - 2x_2)$ then verify that T is linear transformation. Describe null space of T.

7 Prove that a linear transformation E on vector space is a projection if $E^2 = E$.

UNIT - IV

8 If standard basis of a linear transformation T is an $n \times n$ matrix and has n distinct eigen values then prove that T is diagonalizable.

9 State and prove Caley-Hamilton theorem.

UNIT - V

10 If V be a finite dimensional vector space over a field of characteristic zero and f be a symmetric bilinear form on V. Then prove that there is an ordered basis for V in which f is represented by a diagonal matrix.

11 If $\{u_1, u_2 \ldots\ldots u_n\}$ is any finite orthonormal set in an inner product space V and ϑ is any vector in V then prove that

$$\sum_{i=1}^{n} |< \vartheta, u_i >|^2 \leq \|\vartheta\|^2$$

SECTION - C

12 Show that every finite group G has a composition series but an infinite abelian group does not have a composition series.

13 If F is a field and $p(x)$ be an irreducible polynomial of positive degree over a field F, then prove that there is an extension $K = \frac{F(x)}{P(x)}$ of F such that $[K:F] = \deg p(x)$ and $p(x)$ has a root in K.

14 If V be finite dimensional vector space over the field F and W be a subspace of V whose annihilator is the set W^o then prove that $\dim W + \dim W^o = \dim V$.

15 (a) Show that every finite dimensional vector space V with an inner product has an orthonormal basis.

(b) Prove that similar matrices have the same minimal polynomial.

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