U-1653

M. A. / M. Sc. (Previous) Examination, 2021

MATHEMATICS

Paper - III

Partial Differential Equation and Mechanics

Time allowed : **Three Hours**

Maximum Marks : **100**

*This question paper contains **three** sections as under :*

SECTION - A

1 (i) Define complete integral and envelop for P. D. E.

(ii) Using method of separation of varibles, solve $\frac{\partial u}{\partial x} = u + 2\frac{\partial u}{\partial t}$ where $u(x, 0) = 6e^{-3x}$.

(iii) Define principle of superposition for a P. D. E.

(iv) Define radius of gyration for a rigid body about a given line.

(v) Classify the equation

$$\frac{\partial u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(vi) What do you mean by momental Ellipsoids at a point of the rigid body.

(vii) Define Holonomic and no holonomic, scleronomic and rhenomic system.

(viii) What do you mean by ignorable co-ordinates for a dynamical system.

(ix) Define Geodesic.

(x) Write down Harxilton's principle.

https://pyqlelp.com

SECTION - B

UNIT - I

2 Show that the solution $y(x)$ of the IVP $\frac{dy}{dx} = y^2 + \cos x^2 \;\; y(0) = 0$ exists for $|x| \leq \frac{1}{2}$.

3 Reduce the equation $\frac{\partial^2 y}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ canonical form.

UNIT - II

4 Show that the general solution of wave equation $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$ is

$\phi = f(x + ct) + g(x - ct)$.

5 Solve the differential equation

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0$$

With boundary condition

(i) v is finite when $r \to 0$

(ii) $v = \sum c_n \cos n\theta$ when $r = a$.

UNIT - III

6 If A and B be the moments of inertia of a uniform lamina about two perpendicular axes OX and OY lying in its plane and F be the product of inertia of the lamina about these lines, show that principal moments at 0 are equal to

$$\frac{1}{2}\left[(A+B) \pm \sqrt{\left(A - B^2\right) - 4F^2}\right]$$

7 Deduce general equations of motion of a rigid body from De Alembert's principle.

https://pyqlelo.com

UNIT - IV

8 When the Lagrangian function has the form $L = q_k\, q_k - \sqrt{1 - q_k^2}$ show that the generalized acceleration is zero.

9 A body moves about a point O under no forces. The principal moments of inertia at O being $3A$, $5A$ and $6A$ initially the angular velocity has components $w_1 = n, w_2 = 0, w_3 = n$ about the corresponding principal axes. show that at any later time t, $w_2 = \frac{3n}{\sqrt{5}} \tanh \frac{nt}{\sqrt{5}}$ and the body ultimately rotates about its mean axis.

UNIT - V

10 Find the curve joining the two points (x_1, y_1) and (x_2, y_2) that yields a surface of revolution of minimum area, when revolved about the axis.

11 Establish Hamitonis canonical equation of motion.

SECTION - C

12 State and prove existance and uniqueness theorem for the solution of the equation.

$$\frac{dy}{dx} = f(x, y)$$

13 A thin rectangular plate whose surface is impervious to heat flow has at $t = 0$ an arbitrary distribution of temperature $f(x, y)$. It has four edges $x = 0, x = a, y = 0$ and $y = b$ are kept at zero temperature. Determine the temperature at a point of a plate as t increases.

14 (a) Show that the M.I. of a right solid cone, whose height is h and radius of its base is a, is $\frac{3M}{20}\left(a^2 + 4h^2\right)$ about a line passing through the vertex parallel to its base.

(b) A uniform semi circular arc of mass m and radius a, is fixed at its ends to two points in the same vertical line and is rotating with constant angular velocity w. Show that the horizontal thrust on the upper end is $m\left[\frac{g+w^2a}{\pi}\right]$.

15 (a) Discuss the use of Lagrange's equations of motion for the problems of small oscillation.

(b) Use Hamilton's equations to find the equations of motion of a projectile in space.

https://www.uokonline.com