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M.A./M.Sc. (Previous)

Examination, 2019

MATHEMATICS

Paper III

(Partial Differential Equations and Mechanics)

Time Allowed : Three Hours

Maximum Marks : 100

*This question paper contains **three** sections as under :*

Section-A ***Max. Marks-10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*

Section-B ***Max. Marks-50***

*This section contains **10** questions having **2** questions from each unit. Answer **5** questions (**250** words each) selecting **one** question from each unit. **All** questions carry equal marks.*

Section-C ***Max. Marks-40***

*This section contains **4** descriptive type questions (questions may have sub-divisions) covering all units but not more than **one** question from each unit. Answer any **two** questions (**500** words each). **All** questions carry equal marks.*

Section A 10×1=10

1. (*i*) Classify the following P.D.E. :

$$\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial x^2 y} + y\frac{\partial^2 u}{\partial y^2} = 0.$$

(*ii*) Define Cauchy problem.

(*iii*) Write two-dimensional wave equation in Cartesian form.

(*iv*) Write three-dimensional heat equation in Cartesian form.

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(v) Define product of inertia.

(vi) Explain equation of motion (principle of angular momentum).

(vii) Define holonomic and non-holonomic dynamical systems.

(viii) Write Euler's dynamical equations.

(ix) What do you mean by calculus of variations ?

(x) State principle of least action.

Section B

UNIT-I

2. Examine the existence and uniqueness of solution of the initial value problem :

$$\frac{dy}{dx} = y^2,\ y(1) = -1.$$

3. Reduce the following P.D.E. into canonical form and solve it :

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2$$

UNIT-II

4. Solve $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$, where $z = z(x, t)$ with the boundary conditions : $z(0, t) = z(l, t) = 0$ for all t.

5. Solve :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{C^2}\frac{\partial^2 z}{\partial t^2}$$

with boundary conditions : $z(0, y, t) = 0$ and $z(a, y, t) = 0$, $z(x, 0, t) = 0$ and $z(x, a, t) = 0$.

UNIT-III

6. Find the M.I. of a semi-circular lamina of mass M and radius a about a tangent parallel to the bounding diameter.

7. A rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α, show that :

$$\omega^2 = \frac{3}{4}\frac{g}{a\cos\alpha}.$$

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UNIT-IV

8. Deduce Euler's dynamical equations from Lagrange's equations of motion.

9. Derive the principle of conservation of energy with the help of Hamilton's canonical equations.

UNIT-V

10. Find the path on which particle, in the absence of friction, will slide from one fixed point to another point not in the same vertical line in the shortest time under the action of gravity.

11. A particle moves in the xy-plane under the influence of a central force depending only on its distance from the origin. Write the Hamiltonian of the system and Hamilton's equations of motion.

Section C

12. (a) Solve :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{C^2}\frac{\partial^2 u}{\partial t^2}$$

subject to BCs : $u(0, y, t) = u(a, y, t) = u(x, 0, t) = u(x, b, t) = 0$ and ICs : $u(x, y, 0) = f(x, y)$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = g(x, y)$.

(b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. It is released from rest from this position. Find the displacement $y(x, t)$.

13. (a) Show that on extrimity of the bounding diameter of a semi-circular lamina, the principal axes makes an angle $z \tan^{-1}\left(\frac{8}{3n}\right)$ to the diameter.

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(b) A uniform vertical circular plate of radius a, is capable of revolving about a smooth horizontal axis through its centre; a rough perfectly flexible chain, whose mass is equal to that of the plate and whose length is equal to its circumference hangs over its rim in equilibrium. If one end be slightly displaced, show that the velocity of the chain when the other end reaches the plate is :

$$\sqrt{(\pi ag/6)}.$$

14. Obtain Lagrange's equations of motion in generalized coordinates for a holonomic dynamical system. Discuss their use for problems of small oscillation.

15. (a) Find the shape of the plane curve of the given perimeter enclosing maximum area.

(b) A particle of unit mass is projected so that total energy is h, in a field of force of which the potential energy is $\phi(r)$ at a distance r from the origin. Deduce from the principles of energy and least action that the differential equation of the path is :

$$c^2\left\{r^2 + \left(\frac{dr}{d\theta}\right)^2\right\} = r^4\{h - \phi(r)\}.$$

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