M.A./M.Sc. (Previous)
Examination, 2017
MATHEMATICS
Paper III
(Partial Differential Equations and Mechanics)

Time Allowed : Three Hours

Maximum Marks : 100

*This question paper contains **three** sections as under :*

Section-A ***Max. Marks 10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*

Section-B ***Max. Marks-50***

*This section contains **10** questions having **2** questions from each **unit**. Answer **5** questions (**250** words each) selecting **one** question from each unit. **All** questions carry **equal** marks.*

Section-C ***Max. Marks-40***

*This section contains **4** descriptive type questions (question may have sub-divisions) covering all units but not more than one question from each unit. Answer any **two** questions (**500** words each). **All** questions carry equal marks.*

Section A

1. (*i*) Write Lipschitz condition for function $f(x, y)$.

(*ii*) Examine the nature of equation :

$$y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}.$$

(*iii*) Write Laplace's equation in polar co-ordinates in two dimensions.

(*iv*) Write Laplace's equation in spherical polar co-ordinates.

(*v*) State D'Alembert's principle.

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$2a$ and mass m about a line through its centre perpendicular to its length.

(*vii*) Define Holonomic and Non-Holonomic System.

(*viii*) Define generalised co-ordinates.

(*ix*) Distinct between Hamilton's principle and principle of least action.

(*x*) Define Geodesic.

Section B

UNIT-I

2. Reduce the equation in canonical form :

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2.$$

3. Find the complete integral of :

$$(y-x)(qy-px) = (p-q)^2.$$

UNIT-II

4. Explain D'Alembert's solution for one-dimensional wave equation.

5. Explain general solution of one-dimensional heat equation in Cartesian co-ordinates.

UNIT-III

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6. Find the moment of inertia of the triangle ABC about a perpendicular to its plane through A.

7. A uniform rod OA of length $2a$, free to turn about its end O, resolves with uniform angular velocity ω about a vertical OZ through O, and is inclined at a constant angle α to OZ. Prove that α is either zero or $\cos^{-1}(3g/4a\omega^2)$.

UNIT-IV

8. Deduce principle of energy from Lagrange's equations for conservative field.

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9. A body under the action of no forces moves so that the resolved part of its angular velocity about one of the principal axis at the centre of gravity is constant. Prove that the angular velocity of body must be constant.

UNIT-V

10. A particle moves in the xy-plane under the influence of a central force depending only on its distance from the origin :

(a) Set up the Hamiltonian for the system.

(b) Write Hamilton's equation of motion.

Find the extremals of the following functional :

$$\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x)\, dx$$

with $y(0) = 0$ and $y(1) = e$.

Section C

State and prove existence and uniqueness theorem for the solution of equation : 2+18

$$\frac{dy}{dx} = f(x, y).$$

13. A thin rectangular plate whose surface ... impervious to heat flow has at $t = 0$ an arbitrary distribution of temperature $f(x, y)$. Its four edges $x = 0$, $y = 0$, $x = a$, $y = b$ are kept at zero temperature. Determine the temperature at a point of the plate as t increases.

14. (a) A uniform vertical circular plate of radius a is capable of revolving about a smooth horizontal axis through its centre, a rough perfectly flexible chain, whose mass is equal to that of plate and whose length is equal to its circumference hangs over its rim in equilibrium. If one end be slightly displaced show that the velocity of the chain when the other end reaches the plate is $\left(\frac{\pi ag}{6}\right)^{1/2}$. 1

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(b) One end of a light string is fixed to a point of the rim of uniform circular disc of radius a and mass m and the string is wounded serveral times round the rim. The free end is attached to a fixed point and the disc is held so that the part of the string not in contact with it, is vertical. If the disc be let go, find the acceleration and tension of the string. 5

5. (a) Using cylindrical co-ordinates, write Hamiltonian and Hamilton's equation for a particle of mass m moving on the inside of a frictionless cone : 10

$$x^2 + y^2 = z^2 \tan^2 \alpha .$$

(b) Find the shape of the plane curve of fixed length l so that it encloses the maximum area. 10

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