This question paper contains 8 printed pages]

1653

M.A./M.Sc. (Previous)

Examination, 2018

MATHEMATICS

Paper III

(Partial Differential Equations and Mechanics)

Time Allowed : Three Hours

Maximum Marks : 100

*This question paper contains **three** sections as under :*

Section-A ***Max. Marks-10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*

Section-B ***Max. Marks-50***

*This section contains **10** questions having **2** questions from each unit. Answer **5** questions (**250** words each) selecting **one** question from each unit. **All** questions carry equal marks.*

Section-C ***Max. Marks40***

*This section contains **4** descriptive type questions (questions may have sub-divisions) covering all units but not more than **one** question from each unit. Answer any **two** questions (**500** words each). **All** questions carry equal marks.*

Section A

1. (*i*) Define complete integral and envelop for a P.D.E.

   (*ii*) Write Lipschitz condition for function $f(x, y)$.

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(*iii*) Write heat conduction equation.

(*iv*) Write Laplace's equation in polar coordinate in two dimension.

(*v*) Define radius of gyration of a rigid body about a given line.

(*vi*) State De Alembert's principle.

(*vii*) Define generalised co-ordinates.

(*viii*) Define Eulerian angles.

(*ix*) Write down Hamilton's principle.

(*x*) Define Geodesic.

Section B

UNIT-I

2. Show that the solution $y(x)$ of IVP $\frac{dy}{dx} = y^2 + \cos x^2$, $y(0) = 0$ exists for $|x| \leq \frac{1}{2}$.

3. Transform the following P.D.E. into canonical form and hence solve it :

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0.$$

UNIT-II

4. Solve $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ subject to boundary conditions : http://www.uokonline.com

(*i*) $y = 0$ when $x = 0$

(*ii*) $y = 0$ when $x = l$

(*iii*) $\frac{\partial y}{\partial t} = 0$ when $t = 0$ and

(*iv*) $y = \begin{cases} \frac{2kx}{l}, & 0 < x < \frac{l}{2} \\ \frac{2k(l-x)}{l}, & \frac{l}{2} < x < l \end{cases}$

5. Solve the differential equation :

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0 \text{ s.t.}$$

(*i*) V is finite when $r \to 0$

(*ii*) $V = \Sigma C_n \cos(n\theta)$ on $r = a$.

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UNIT-III

6. Find the M.I. of a truncated cone about its axis, the radii of its end being a and b.

7. Deduce general equations of motion of a rigid body from the De Alembert's principle.

UNIT-IV

8. Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis.

9. A body under the action of no forces moves so that the resolved part of its angular velocity about one of the principal axes at the centre of gravity is constant. Show that the velocity of the body must be constant.

UNIT-V

10. Prove that the sphere is the solid figure of revolution which, for a given surface area, has maximum volume.

11. Establish Hamilton's canonical equation of motion.

Section C

12. State and prove existence and uniqueness theorem for the solution of the equation :

$$\frac{dy}{dx} = f(x, y).$$

13. (*a*) Solve :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

satisfying the B.Cs

$u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\,\pi x}{l}\right)$.

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(b) Heat flows in a semi-infinite rectangular plate, the ends $x = 0$ being kept at temperature u_0 and the long edges $y = 0$ and $y = a$ at zero temperature. Prove that temperature at any point (x, y) is

$$\frac{4u_0}{\pi} \sum_{m=0}^{\infty} \frac{1}{(2m+1)} \sin \frac{(2m+1)\pi y}{a} e^{-(2m+1)\frac{\pi x}{a}}.$$

14. (a) A uniform semi-circular arc, of mass m and radius a, is fixed at its ends to two points in the same vertical line and is rotating with constant angular velocity ω. Show that the horizontal thrust on the upper end is $m\left(\frac{g + \omega^2 a}{\pi}\right)$.

(b) Deduce principle of energy from Lagrange's equations of motion.

15. (a) A sphere rolls down a rough included plane, if x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration.

(b) Show that the geodesics on a plane are straight line.

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