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M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper III

(Partial Differential Equations and Mechanics)

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This section contains ***one*** *compulsory question with* ***10*** *parts, having* ***2*** *parts from each unit, short answer in* ***20*** *words for each part.* ***All*** *questions carry equal marks.*

Section-B ***Max. Marks-50***

This section contains ***10*** *questions having* ***2*** *questions from each unit. Answer* ***5*** *questions (****250*** *words each) selecting* ***one*** *question from each unit.* ***All*** *questions carry equal marks.*

Section-C ***Max. Marks-40***

This section contains ***4*** *descriptv type questions (questions may have sub-divisions) covering all units but not more than one question from each unit. Answer any* ***two*** *questions. (****500*** *words each).* ***All*** *questions carry equal marks.*

Section A

1. (i) Find the envelope of the family of spheres of unit radius with centres on x-axis.

(ii) Define Lipschitz constant.

(iii) Write two-dimensional wave equation in polar coordinates.

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(iv) Write three-dimensional diffusion equation in Cartesian coordinates.

(v) Define principal axes.

(vi) Define moment of inertia.

(vii) Define generalized coordinates.

(viii) Define conservative or non-conservative dynamical systems with an example.

(ix) Define an isoperimetric problem.

(x) Define an Euler-Lagrange differential equation for an extremal.

Section B

UNIT-I

2. Find the complete integral of the equation $Z^2 = pqxy$.

Or

3. Solve the Cauchy's problem for $zp + q = 1$, when initial data curve is $x_0 = \mu$, $y_0 = \mu$, $z_0 = \mu/2$, $0 \le \mu \le 1$.

UNIT-II

4. By separating the variables, show that the one-dimensional wave equation :

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$$

has solution of the form A $\exp(\pm inx \pm inct)$ where A and n are constants.

Or

5. Obtain a solution of Laplace's equation in rectangular Cartesian coordinates (x, y, z) by the method of separation of variables.

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UNIT-III

6. A solid homogeneous cone of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4 + \tan^2\alpha)$.

Or

7. If A and B be the moments of inertia of a uniform lamina about two perpendicular axes OX and OY, lying in its plane, and F be the product of inertia of the lamina about these lines, show that the principal moments at O are equal to :

$$\frac{1}{2}[A + B \pm \sqrt{(A - B)^2 + 4F^2}].$$

UNIT-IV

8. Define Hamiltonian function. Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H is the Hamiltonian function.

Or

9. When the Lagrangian function has the form :

$$L = \dot{q}_k q_k - \sqrt{1 - \dot{q}_k^2}$$

show that the generalized acceleration is zero.

UNIT-V

10. On what curves can the functional :

$$I[y(x)] = \int_0^1 \{(y')^2 + 12xy\}\, dx$$

$$y(0) = 0,\ y(1) = 1$$

be extremized ?

Or

11. Show that the curve of shortest distance on a right circular cylinder is a helix.

Section C

12. Reduce the following equation to canonical form and hence solve it :

$$y(x + y)(r - s) - xp - yq - z = 0.$$

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13. Solve the two-dimensional wave equation :

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

by the method of separation of variables subject to the conditions :

(*i*) $y = 0$ when $x = 0$

(*ii*) $y = 0$ when $x = l$

(*iii*) $\frac{\partial y}{\partial t} = 0$ when $t = 0$ and

(*iv*) $y = \begin{cases} \frac{2k}{l}x & 0 < x < l/2 \\ \frac{2k}{l}(l-x) & \frac{1}{2} < x < l \end{cases}$ when $t = 0$.

14. A rod, of length $2a$, is suspended by a string, of length l, attached to one end; if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that :

$$3l(\tan\phi - \tan\theta)\sin\theta = (4\tan\theta - 3\tan\phi)\, a\sin\phi$$

15. Find the closed convex curve of length L that encloses the greatest possible area.

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