UB-1653

M. A. / M. Sc. (Previous) Examination, 2023

MATHEMATICS

Paper - III

Partial Differential Equations and Mechanics

Time allowed : **Three Hours**

Maximum Marks : **100**

This question paper contains three sections as under :

Section-A खण्ड-अ **Max. Marks 10**

This section contains one compulsory question with 10 parts, having 2 parts from each unit. Answer in 20 words for each part. All questions carry equal marks.

इस खण्ड में एक अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से दो लघु प्रश्न लेते हुये कुल दस लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर 20 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान हैं।

Section-B खण्ड-ब **Max. Marks 50**

This section contains 10 questions having 2 questions from each unit. Answer 5 questions (250 words each) selecting one question from each unit. All questions carry equal marks.

इस खण्ड में प्रत्येक इकाई से दो दो प्रश्न लेते हुये कुल दस प्रश्न हैं। प्रत्येक इकाई से एक प्रश्न का चयन करते हुये कुल 5 प्रश्नों के उत्तर देने हैं। प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान हैं।

Section-C खण्ड-स **Max. Marks 40**

This section contains 4 descriptive type questions (questions may have sub-parts) covering all units but not more than one question from each unit. Answer any four questions (500 words each). All questions carry equal marks.

इस खण्ड में 4 वर्णनात्मक प्रश्न होंगे। प्रश्न में भाग भी हो सकते हैं जो सभी इकाईयों में से दिये जायेंगे किन्तु एक इकाई में से एक से अधिक प्रश्न नहीं होगा। कोई चार प्रश्नों के उत्तर दिये जाने हैं। प्रत्येक प्रश्न का उत्तर 500 शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान हैं।

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SECTION - A

1 (i) Write Lipschitz condition for a function $f(x, y)$.

(ii) Examine the nature of the equation

$$y^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + 4u = 0$$

(iii) Write Laplace's equation in polar coordinates.

(iv) Write heat equation in three dimension.

(v) Find M.I. of a rod of length $2a$ and mass m about a line through its edge perpendicular to its length.

(vi) State D'Alembert's principle.

(vii) Define Holonomic system.

(viii) Explain generalized coordinates.

(ix) Define "Functional".

(x) Define Geodesic.

SECTION - B

UNIT - I

2 Reduce the equation in canonical form:

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2$$

3 Show that the solution $y(x)$ of I.V.P.

$\frac{dy}{dx} = y^2 + \cos x^2$, $y(0) = 0$ exists for $|x| \leq \frac{1}{2}$

UNIT - II

4 Explain D'Alembert's solution for one dimensional wave equation.

5 Solve the differential equation

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{r\theta^2} = 0 \text{ s.t.}$$

(i) V is finite when $r \to 0$

(ii) $V = \sum C_n \cos(n\theta)$ on $r = a$

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UNIT - III

6 Find the M.I. of truncated cone about its axis, the radii of its ends being a and b. $(a > b)$.

7 A rough uniform board of mass m and length 2a, rests on a smooth horizontal plane and a man of mass M walks on it from one end to other. Find the distance through which the board moves in this time.

UNIT - IV

8 Define Lagrangian function. When the Lagrangian function has the form $L = \dot{q}_k q_k - \sqrt{1 - \dot{q}_k^2}$ show that the generalized acceleration is zero.

9 Derive Hamilton's Canonical equations of motion.

UNIT - V

10 Find the path on which a particle, in the absence of friction, will slide from one fixed point to another point in the shortest time under the action of gravity.

11 Find the extremal for the functional

$$\int_1^2 \frac{\sqrt{1+y'^2}}{x}\,dx, \quad y(1) = 0,\ y(2) = 1$$

SECTION - C

12 (a) Examine the existence and uniqueness of the solution of the following equation.

$$\frac{dy}{dx} = y^2 \ ; \ y(1) = -1$$

(b) Solve

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \ ; \ u(0, y) = 4e^{-y} - e^{-5y}$$

13 (a) A thin rectangular plate whose surface is impervious to heat flow has at $t=0$ an arbitrary distribution of temperature $f(x, y)$. Its four edges $x = 0$, $x = a$, $y = 0$ and $y = b$ are kept at zero temperature. Determine the temperature at a point of the plate as t increases.

(b) A uniform semicircular arc, of mass m and radius a is fixed at its end points in the same vertical line, and is rotating with constant angular velocity w. Show that the horizontal thrust on the upper end is $m\left(\frac{g + w^2 a}{\pi}\right)$.

14 (a) Describe three Eulerian angles, when a body can rotate freely about a fixed point.

(b) Define Hamilton's principle. Use Hamilton's equations to find the equations of motion of a projectile in space.

15 (a) State and prove principle of least action.

(b) Show that Geodesics on a plane are straight lines.

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