[This question paper contains 8 printed pages]

1652

M.A./M.Sc. (Previous) Examination, 2018

MATHEMATICS

Paper II

(Real and Complex Analysis)

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains three sections as under :

Section-A **खण्ड 'अ'**

This section contains one compulsory question with 10 parts, having 2 parts from each unit, short answer in 20 words for each part. All questions carry equal marks.

इस खण्ड में एक अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से 2 लघु प्रश्न लेते हुए कुल 10 लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर 20 शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान हैं।

This section contains 10 questions having 2 questions from each unit. Answer 5 questions (250 words each) selecting one question from each unit. All questions carry equal marks.

इस खण्ड मे प्रत्येक इकाई से 2 प्रश्न लेते हुए कुल 10 प्रश्न हैं। प्रत्येक इकाई से एक प्रश्न का चयन करते हुए, कुल 5 प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान हैं।

Section-B **खण्ड 'ब'** **Max. Marks : 40**

This section contains 4 descriptive type questions (Questions may have sub-division) covering all units but not more than one question from each unit. Answer any two questions (500 words each). All questions carry equal marks.

इस खण्ड में 4 वर्णनात्मक प्रश्न होंगे (प्रश्न के उप-भाग भी हो सकते हैं) जो सभी इकाइयों में से दिये जायेंगे, किन्तु एक इकाई से एक से अधिक प्रश्न नहीं होगा। कोई दो प्रश्नों के उत्तर दिये जाने हैं। प्रत्येक प्रश्न का उत्तर 500 शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान हैं।

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Section A

1. (*i*) Define Riemann-Stieltjes Integral as a limit of sums.

(*ii*) Define pointwise convergence of a sequence.

(*iii*) Define Lebesgue outer measure.

(*iv*) Define the convergence in measure of a sequence of measurable functions.

(*v*) Define Bilinear transformation.

(*vi*) Define circle of convergence of power series.

(*vii*) Define contour.

(*viii*) Define multiconnected region.

(*ix*) Define Removable singularity.

(*x*) Define Analytic Continuation.

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UNIT-I

2. Let $f(x) = x$ and $g(x) = x^2$ are defined on $[0, 1] \to R$. Does $\int_0^1 f\,dg$ exists ? If it exists, find its value. http://www.uokonline.com

3. Let $\langle f_n \rangle$ be a sequence of function defined on $[a, b]$. Let $\lim_{n \to \infty} f_n(x) = f(x)$ for all $x \in [a, b]$ and let $M_n = \sup\{|f_n(x) - f(x)| : x \in [a, b]\}$. Then sequence $\langle f_n(x) \rangle$ converges uniformly to $f(x)$ if and only if $M_n \to 0$ as $n \to \infty$.

UNIT-II

4. Let $\{E_n\}$ be a countable collection of sets. Then prove that :

$$m^*\left(\bigcup_n E_n\right) \leq \sum_n m^*(E_n).$$

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5. Let f and g be measurable functions defined on a measurable set E, then $f + g$ and fg are measurable functions on E. Prove it.

UNIT-III

6. Show that we may identify by stereographic projection, the complex plane with the sphere

$$x^2 + y^2 + \left(z - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2.$$

Find the image of the point $z = x + iy$ on the above sphere.

7. Discuss the transformation $\omega = z^2$. Find the image of the hyperbola $x^2 - y^2 = c$ under this transformation.

UNIT-IV

8. If $f(z)$ is analytic in a region corresponding the disk C : $|z| \leq R$. Then for $z = re^{i\theta}$, $r < R$ we have :

$$f(re^{i\theta}) = \frac{1}{2\pi} \int_0^{2\pi} \frac{(R^2 - r^2) f(Re^{i\phi})}{(R^2 - 2Rr\cos(\theta - \phi) + r^2)} d\phi.$$

9. Expand the function $\frac{z^2 - 4}{(z+1)(z+4)}$ which are valid for the following regions :

(i) $|z| < 1$

(ii) $1 < |z| < 4$

(iii) $|z| > 4$.

UNIT-V

10. Find the residue of $\frac{z^2 - 2z}{(z+1)^2 (z^2 + 4)}$ at all its poles in the finite plane.

11. Prove that every polynomial of degree $n \geq 1$ has at least one zero.

Section C

12. (a) Let f, g a... RS-integrable on $[a, b]$, then their sum $f + g$ is also RS-integrable on $[a, b]$ and :

$$\int_a^b (f + g)d\alpha = \int_a^b f d\alpha + \int_a^b g d\alpha.$$

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$$f_n(x) = nx(1 - x)^n$$

does not converge uniformly on [0, 1]

13. (a) Let $\{E_n\}$ be an increasing sequence of measurable sets, then :

$$m\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n \to \infty} m(E_n).$$

Prove it.

(b) Let $\langle f_n \rangle$ be a sequence of measurable functions converges pointwise to a function f on E. $\left\{i.e. \lim_{n \to \infty} f_n(x) = f(x)\right\}$, then the function f is measurable.

14. (a) If $u + v = \dfrac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ and $f(z) = u + iv$ is an analytic function, then find $f(z)$ in terms of $f(z)$.

(b) Prove that if $f(z)$ is analytic in a simply connected domain D, then the indefinite integral $\int_{z_0}^{z} f(z)dz$ is independent of the path joining z_0 with z in D.

15. (a) Let $f(z)$ be a continuous function in a simply connected domain D, if $\int_C f(z)dz = 0$ along every simple closed contour C. Then prove that $f(z)$ is analytic in D.

(b) Prove that :

$$\int_0^{2\pi} \frac{d\theta}{1 + a^2 - 2a\cos\theta} = \frac{2\pi}{1 - a^2},$$

$0 < a < 1$.

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