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1652

M.A./M.Sc. (Previous) Examination, 2019

MATHEMATICS

Paper II

(Real and Complex Analysis)

Time Allowed : Three Hours

Maximum Marks : 100

*This question paper contains **three** sections as under :*

Section-A **खण्ड 'अ'** ***Max. Marks-10***

*This section **contains one** compulsory question **with 10** parts, having **2** parts from each unit, short **answer in 20** words for each part. **All** questions carry **equal** marks.*

***इस खण्ड में एक** अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई **से 2 लघु प्रश्न** लेते हुए **कुल 10** लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न **का उत्तर 20** शब्दों से **अधिक** न हो। **सभी** प्रश्नों के अंक समान*

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Section-B **खण्ड 'ब'** ***Max. Marks-50***

This section contains 10** questions having 2 **questions from each unit. Answer 5** questions (250 **words each) selecting one** question from each unit. **All questions carry equal marks.

***इस खण्ड में प्रत्येक** इकाई से **2** प्रश्न लेते हुए कुल 10 **प्रश्न हैं। प्रत्येक** इकाई से **एक** प्रश्न का चयन करते हुए, कुल 5 **प्रश्नों** के उत्तर दीजिए। **प्रत्येक** प्रश्न का उत्तर **250** शब्दों से अधिक **न हो। सभी** प्रश्नों के अंक समान हैं।*

Section-C **खण्ड 'स'** ***Max. Marks-40***

This section contains 4 descriptive type** questions **(questions may have sub-divisions) covering all** units **but not more than one question from each unit.** Answer **any two questions (500** words each). **All questions** carry **equal marks.

इस खण्ड में 4** वर्णनात्मक प्रश्न होंगे (प्रश्नों के उप-भाग **भी हो सकते हैं) जो सभी** इकाइयों में से दिये जायेंगे, किन्तु एक **इकाई से एक से अधिक** प्रश्न नहीं होगा। किन्हीं **दो** प्रश्नों के उत्त **दिये जाने हैं। प्रत्येक** प्रश्न का उत्तर **500** शब्दों से अधिक न हो **सभी प्रश्नों के अंक समान हैं।

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Section A

1. (*i*) Let f and α be bounded functions on $[a, b]$ and α be monotonic increasing on $[a, b]$, then define Riemann-Stieltjes upper integral of f with respect to α.

(*ii*) Define uniform convergence of a series of functions.

(*iii*) Define measurable set.

(*iv*) Define measurable function.

(*v*) Write polar form of Cauchy-Riemann equations.

(*vi*) Find the fixed points of the transformation :

$$\omega = \frac{(2+i)z - 2}{z - i}.$$

(*vii*) Evaluate

$$\int_C |dz|$$

where C is any rectifiable arc joining the points $z = a$ and $z = b$. 3

(*viii*) Write the value of the integral :

$$\int_C z^2\, dz;$$

where C is the circle $|z - a| = R$.

(*ix*) Find the singularity of the function $\frac{e^2 - 1}{z}$.

(*x*) Write Cauchy's residue theorem.

Section B

UNIT-I

2. Prove that a function f is Riemann-Stieltjes integrable with respect to α an $[a, b]$ if and only

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if for every $\varepsilon > 0$, there exists a partition P of $[a, b]$ such that :

$$U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon.$$

3. Prove that the sum function of a uniform convergent series $\sum u_n(x)$ of functions whose each term is continuous in an interval $[a, b]$ is also continuous in the interval $[a, b]$.

UNIT-II

4. Let $\{E_1, E_2, \ldots\ldots, E_n\}$ be a finite class of disjoint measurable sets, then prove that for any set A :

$$m^*\left(A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*(A \cap E_i).$$

5. Let $\langle f_n \rangle$ be a sequence of measurable functions convergence in measure to two functions f and g, then prove that these limit functions are equivalent.

[Contd....

UNIT-III

6. Prove that the function $u(x, y) = x^3 - 3xy^2$ is harmonic and obtain its conjugate $v(x, y)$ such that $f(x, y) = u(x, y) + iv(x, y)$ is analytic.

7. Show that both the transformations :

$$\omega = \frac{1 - z}{1 + z} \text{ and } \omega = \frac{z - 1}{z + 1}$$

Transform $|\omega| \leq 1$ into the half plane $Re(z) \geq 0$.

UNIT-IV

8. State and prove Cauchy's integral formula for simply connected domain.

9. State and prove maximal modulus principle.

UNIT-V

10. If $f(z)$ has an isolated essential singularity a $z = a$, then prove that $f(z)$ comes arbitrary clos to every complex value in each delete neighbourhood of a.

[Contd..

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11. Prove that the power series

$$z + \frac{1}{2}z^2 + \frac{1}{3}z^3 + \ldots\ldots$$

and

$$i\pi - (z-2) + \frac{1}{2}(z-2)^2 - \ldots\ldots$$

has no common region of convergence. Nevertheless they are analytic continuation of the same function.

Section C

12. (*a*) Prove that the following series is uniformly convergent for all values of $x \geq 0$:

$$\frac{1}{1+x^2} - \frac{1}{2+x^2} + \frac{1}{3+x^2} - \ldots\ldots$$

(*b*) Prove that the bilinear transformation transforms the circle $\arg \frac{z-z_1}{z-z_2} = \lambda$ into a similar circle $\arg \frac{w-w_1}{w-w_2} = \text{constant}$, where w_1, w_2 correspond to z_1, z_2 respectively.

13. (*a*) Let $\{E_n\}$ be an increasing sequence of measurable sets, then prove that :

$$m\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m(E_n).$$

(*b*) Let f be a bounded measurable function defined on a measurable set E, then :

$$\left|\int_E f(x)dx\right| \leq \int_E |f(x)|dx.$$

14. (*a*) State and prove Morera's theorem.

(*b*) State and prove Liouville's theorem.

(*a*) State and prove Rouche's theorem.

(*b*) Evaluate :

$$\int_0^{\infty} \frac{dx}{x^6+1}.$$

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