This question paper contains 4+2 printed pages]

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M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper II

(Real and Complex Analysis)

Time Allowed : Three Hours

Maximum Marks : 100

*This question paper contains **three** sections as under :*

Section-A ***Max. Marks-10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*

Section-B ***Max. Marks-50***

*This section contains **10** questions having **2** questions from each unit. Answer **5** questions (**250** words each) selecting **one** question from each unit. **All** questions carry equal marks.*

Section-C ***Max. Marks-40***

*This section contains **4** descriptive type questions (questions may have sub-division) covering all units but not more than one question from each unit. Answer any **two** questions (**500** words each). **All** questions carry equal marks.*

Section A **10×1=10**

1. (*i*) Define the upper-Riemann-Stieltjes integral.

   (*ii*) Write Weierstrass's M-test for uniform convergence.

(iii) Define Borel measurable.

(iv) Define Lebesgue integral function.

(v) Define power series.

(vi) Write transformation which are Isogonal but not conformal.

(vii) Evaluate $\int_C |dz|$, where C is any rectifiable are joining the points $z = a$ and $z = b$.

(viii) State Laurent's theorem.

(ix) Find the singularity of the function $\frac{e^z - 1}{z}$.

(x) State Cauchy's residue theorem.

Section B 10×5=50

UNIT-I

2. If $f(x) = k$ be a constant function defined on $[a, b]$ and $\alpha(x)$ is a monotonically non-decreasing function on $[a, b]$, then prove that $f(x)$ is Riemann-Stieltjes integrable.

Or

3. Show that the series $\sum \frac{(-1)^{n-1}}{n} |x|^n$ is uniformly convergent on $[-1, 1]$.

UNIT-II

4. Prove that a continuous function defined on a measurable set is always measurable.

Or

5. Let f be a bounded measurable function on a measurable set E, then prove that :

$$\left| \int_E f(x)\,dx \right| \leq \int_E |f(x)|\,dx$$

UNIT-III

6. Derive the Polar form of Cauchy-Riemann equations.

Or

7. Find the image of the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $\omega = 1/z$

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UNIT-IV

8. State and prove maximal modulus principle.

Or

9. Obtain expansion for $\frac{z^2-4}{(z+1)(z+4)}$.

Which are valid, for the regions :

(a) $|z| < 1$

(b) $1 < |z| < 4$

(c) $|z| > 4$.

UNIT-V

10. State and prove Casorati-Weierstrass theorem.

Or

11. Find out the zeros and discuss the nature of singularities of :

$$f(z) = \frac{z-2}{z^2}\sin\frac{1}{z-1}.$$

Section C

12. If f_1 and f_2 are Riemann-Stieltjes integral with respect to $\alpha(x)$ on $[a, b]$. Then $f_1 + f_2$ is Riemann-Stieltjes integral with respect to $\alpha(x)$ on $[a, b]$ and $\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2\, d\alpha$.

13. Prove that the outer measure of an interval is its length.

14. (a) State and prove Morera's theorem.

(b) State and prove Lioville's theorem.

15. (a) State and prove Rouche's theorem.

(b) Find the value of :

$$\int \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz, \; |z| = 1.$$

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