U-1652

M. A. / M. Sc.
(Previous) Examination, 2021

MATHEMATICS

Paper - II

Real & Complex Analysis

Time allowed : **Three Hours**

Maximum Marks : **100**

Note : This question paper contains three sections as under :

SECTION - A

1 (i) Define Riemann-Stieltjes integral.
(ii) Define point wise convergene of a sequence of function.
(iii) Define Borel Set.
(iv) Define lebesgue outer measure.
(v) Define Analytic function.
(vi) Define Conformal mapping.
(vii) State Morera's Theorem.
(viii) Define Schwarz's lemma.
(ix) State Roche's Theorem.
(x) State Casoratti-Weirstrass Theorem.

10×1=10

SECTION - B

UNIT - I

2 Let f be R-integrable functionon [a, b] and let g be a monotonically non-decreasing function on [a, b] such that its derivative g' is R-integrable on [a, b], then f is Riemann-Stieltjes integrable on [a, b] and

$$\int_a^b f dg = \int_a^b f(x)g'(x)dx$$

10

OR

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3 Test for uniform convergence of the sequence and continuity of the limit function f :

$< f_n(x) = n^2 x (1-x)^n \mid x \in R >$ and $\forall n \in N$

10

UNIT - II

4 P.T. if E_1 and E_2 are measurable sets, then $E_1 \cup E_2$ is also measurable.

10

OR

5 Every bounded measurable function f on a measurable set E is l-integrable on E.

10

UNIT - III

6 Show that the funtion f defined by

$f(z) = e^{-z^{-4}}$, $z \neq 0$, $f(0) = 0$ is not analytic at z=0, althour Couchy-Reimann equation are satisfied.

10

OR

7 Determine all bilinear transformations which map the circle $|\,|z| \leq p$ into the circle $|w| \leq p'$.

10

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UNIT - IV

8 P.T. if a function f(z) is analytic for all finite values of z and is bounded then it is a constant.

10

OR

9 Obtain expansion for $\frac{z^2-4}{(z+1)(z+4)}$ which are valid for the region :

(i) $|z| < 1$

(ii) $1 < |z| < 4$

(iii) $|z| > 4$

10

UNIT - V

10 Discuss the nature of singularities of the function.

$$f(z) = \frac{e^{c/(z-a)}}{e^{z/a} - 1}$$

10

OR

11 Show that the function $f(z)=1+z+z^2+z^3+.....$ can be continued analytically outside the cirlce of convergence. **10**

SECTION - C **20×2=40**

12 (i) Prove that the family M of all measurable sets is a σ algebra of sets in P(R).

(ii) Verify the Lebesgue bounded convergence theorem for

$$f_n(x)=\frac{nx}{1+n^2x^2},\ 0\le x\le 1$$

13 (i) State and prove maximum modulus theorem.

(ii) State and prove poisson integral formula.

14 (i) Use Rouche's theorem to show that the equation $z^5+15z+1=0$ has one root in the disk $|z|<3/2$ and four roots in the annulus $3/2<|z|<2$.

(ii) Every polynominal of degree $n\ge 1$ has at least one zero.

15 (i) Show that the transfomation $w(z+i)^2=1$ maps the interior of the circle $|z|=1$ in the z-plane on the exterior of the parabola $\frac{1}{\rho}=2(1-\cos\phi)$ where $w=\rho e^{i\phi}$ in the w-plane

(ii) Find radii of convergence of the following power series

(a) $\sum\frac{(-1)^n}{n}(z-2i)^n$

(b) $\sum\frac{(n+1)}{(n+2)(n+3)}$

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