This question paper contains 8 printed pages]

1654

M.A./M.Sc. (Previous)

Examination, 2018

MATHEMATICS

Paper IV

(Special Functions and Integral Equations)

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains three sections as under :

Section-A ***Max. Marks-10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in 20 words for each part. All questions carry equal marks.*

*This section contains **10** questions having **2** questions from each unit. **Answer 5** questions (**250** words each) selecting **one** question from each unit. **All** questions carry equal marks.*

Section-C ***Max. Marks40***

*This section contains **4** descriptive type questions (questions may have sub-divisions) covering all units but not more than **one** question from each unit. Answer any **two** questions (**500** words each). **All** questions carry equal marks.*

Section A

1. (*i*) State Gauss formula for ${}_2F_1$ with unit argument. 1

(*ii*) State Saalschutz theorem. 1

(*iii*) Define generating function. 1

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(iv) Define confluent hypergeometric function. 1

(v) Define Volterra integral equation of first kind. 1

(vi) Verify whether the function $g(x) = 1$ is a solution of the Fredholm integral equation : 1

$$g(x) + \int_0^1 x\,(e^{xt} - 1)\,g(t)dt = e^x - x.$$

(vii) For the integral equation : 1

$$g(x) = f(x) + \lambda \int_a^x k(x, t)\, g(t)\, dt$$

define the iterated kernel $k_n(x, t)$.

(viii) Find $k_2(x, t)$ for the following integral equation : 1

$$g(x) = f(x) + \lambda \int_0^1 (1 - 3xt)\, g(t)\, dt.$$

(ix) Define Laplace transform. 1

(x) Write change of scale property for inverse Laplace transform. 1

Section B

UNIT-I

2. If $|z| < 1$ and $|z/(1 - z)| < 1$, then prove that 10

$$F\left[\begin{matrix} a, b; \\ c; \end{matrix} z\right] = (1 - z)^{-a} F\left[\begin{matrix} a, c - b; \\ c; \end{matrix} \frac{-z}{1 - z}\right].$$

3. For $F = F(a, b, c, z)$, prove the following contiguous relations : 5+5

(i) $(a - b)\, F = aF(a+) - bF(b+)$

(ii) $(a - c + 1)\, F = aF(a+) - (c - 1)\, F(c-)$.

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4. If $\mathrm{Re}(b) > \mathrm{Re}(a) > 0$, then prove that 10

$$ {}_1F_1(a; b; z) = \frac{\Gamma(b)}{\Gamma(a)\,\Gamma(b-a)} \int_0^1 e^{zt}\, t^{a-1} (1-t)^{b-a-1}\, dt $$

5. For the generating relation : 10

$$ G(2xt - t^2) = \sum_{n=0}^{\infty} g_n(x) t^n $$

Prove that :

$$ g_0(x) = 0 \text{ and for } n \geq 1 $$

$$ x g_n'(x) - n g_n(x) = g_{n-1}(x). $$

UNIT-III

6. Solve the integral equation : 10

$$ g(x) = e^x + \lambda \int_0^1 2e^x e^t\, g(t)\, dt. $$

7. Solve : 10

$$ g(x) = 1 + \int_0^1 (1 + e^{x+t}) g(t)\, dt. $$

UNIT-IV

8. Using resolvent Kernel method, solve : 10

$$ g(x) = \frac{3}{2}e^x - \frac{1}{2}xe^x - \frac{1}{2} + \frac{1}{2}\int_0^1 t\, g(t)\, dt. $$

9. Using the method of successive approximations, solve the integral equation : 10

$$ g(x) = 1 + \int_0^x (x - t)\, g(t)\, dt, \text{ taking } g_0(x) = 0. $$

UNIT-V

10. Prove that : 7+3

$$ L\left\{\mathrm{erf}\sqrt{x}\right\} = \frac{1}{p\sqrt{p+1}} $$

Hence deduce

$$ L\left\{\mathrm{erf}_c\sqrt{x}\right\} = \frac{1}{\sqrt{(p+1)}\,\{\sqrt{(10+1)}+1\}}. $$

11. Using the convolution theorem find the inverse Laplace transform of : 10

$$ \frac{p}{(p^2 + a^2)^2}. $$

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12. Prove that :

$$ {}_3F_2\left[\begin{matrix} a, b, c; \\ 1+a-b,\ 1+a-c; \end{matrix} 1\right] $$

$$ = \frac{\Gamma\left(1+\frac{1}{2}a\right)\Gamma(1+a-b)\,\Gamma(1+a-c)\,\Gamma\left(1+\frac{1}{2}a-b-c\right)}{\Gamma(1+a)\,\Gamma\left(1+\frac{1}{2}a-b\right)\Gamma\left(1+\frac{1}{2}a-c\right)\Gamma(1+a-b-c)}. $$

13. (*i*) For $t \neq 0$ and for all finite z, prove that :

$$ \exp\left[\frac{1}{2}z\left(t-\frac{1}{t}\right)\right] = \sum_{n=-\infty}^{\infty} J_n(z)t^n. $$

(*ii*) Prove that :

$$ xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x). \qquad 14+6 $$

14. Show that the integral equation :

$$ g(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\, g(t)\, dt $$

possesses no solution for $f(x) = x$, but it possesses infinitely many solutions for $f(x) = 1$. 20

15. Solve :

$$ (D^3 - D^2 - D + 1)y = 8te^{-t}, $$

given that :

$$ y(0) = 0,\ y'(0) = 1,\ y''(0) = 0. \qquad 20 $$

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