U-1654

M. A. / M. Sc.
(Previous) Examination, 2021

MATHEMATICS

Paper - IV

Special Functions & Integral Equations

Time allowed : **Three Hours**

Maximum Marks : **100**

This question paper contains ***three*** *sections as under :*

SECTION - A

1 (a) Define Gauss's Hypergeometric function.

(b) State Saalschut's theorem.

(c) Define Bessel's function.

(d) Define orthogonality property of Bessel functions.

(e) Give generating function for Hermite polynomial.

(f) Define Fredholm integral equation of II kind.

(g) Define Neumann series of Kernel $k(x, t)$.

(h) What is orthogonality of the kernels ?

(i) Define Laplace transform.

(j) State the convolution theorem of Laplace transform.

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SECTION - B

UNIT - I

2 Show that if $\mathrm{Re}(b) > 0$ and if n is a non-negative integer,

$$F\left[\begin{matrix} -\frac{1}{2}n, -\frac{1}{2}n+\frac{1}{2}; \\ b+\frac{1}{2}; \end{matrix} 1\right] = \frac{2^n (b)_n}{(2b)_n}.$$

3 Show that if a, b and c are so restricted that each of the functions involved exists; then

$${}_3F_2\left[\begin{matrix} a, b, c; \\ 1+a-b, 1+a-c; \end{matrix} 1\right]$$

$$= \frac{\Gamma\left(1+\frac{a}{2}\right)\Gamma(1+a-b)\,\Gamma(1+a-c)\,\Gamma\left(1+\frac{a}{2}-b-c\right)}{\Gamma(1+a)\,\Gamma\left(1+\frac{1}{2}a-b\right)\Gamma\left(1+\frac{1}{2}a-c\right)\Gamma(1+a-b-c)}$$

UNIT - II

4 Show that if b is neither zero nor a negative integer, ${}_1F_1(a; b; z) = e^z\,{}_1F_1(b-a; b; -z)$.

5 Prove that $x P_n'(x) - n P_n(x) = P_{n-1}'(x)$.

UNIT - III

6 Solve : $g(x) = 1 + \lambda \int_0^{\pi/2} \cos(x-t)\, g(t)\, dt$.

7 Solve : $g(x) = \cos x - x - 2 + \int_0^x (t-x)\, g(t)\, dt$.

UNIT - IV

8 Find the iterated Kernel :

$K(x, t,) = e^x \cos t\,;\ a = 0,\ b = \pi$

9 Solve the integral equation by successive approximation method :

$$g(x) = f(x) + \lambda \int_0^1 e^{x-t}\, g(t)\, dt$$

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UNIT - V

10 If $f(t)$ be continuous for all $t \geq 0$ and be of exponential order as $t \to \infty$ and $f(t)$ is of class A, then show that

$$\lim_{t \to 0} F(t) = \lim_{p \to \infty} pf(p)$$

11 Prove that

(i) $$\int_0^\infty \frac{e^{-at} - e^{-bt}}{t} dt = \log \frac{b}{a}$$

(ii) $$\int_0^\infty tJ_n(t)\, dt = 1$$

SECTION - C

12 An infinitely long string having one end at $x = 0$ is initially at rest on the x-axis. The end $x = 0$ undergoes a periodic transverse displacement given by $A_0 \sin \omega t, t > 0$. Find the displacement of any point on the string at any time.

13 Solve the integral equation :

$$g(x) = e^x - \frac{e}{2} + \frac{1}{2} + \frac{1}{2}\int_0^1 g(t)\, dt$$

14 Solve the volterra equation of I kind :

$$\int_0^x a^{x-t} g(t)\, dt = f(x); f(0) = 0$$

15 Prove that if $\mathrm{Re}(z) < 0$ and if no a_m or b_j is zero or a negative integer,