This question paper contains 7 printed pages]

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M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper – III

(Optional)

MATHEMATICAL STATISTICS

Time Allowed : Three Hours

Maximum Marks : 100

Note : *Non-programmable scientific calculator is allowed in this paper.*

This question paper contains ***three*** *sections as under :*

Section-A **खण्ड 'अ'** ***Max. Marks-10***

This section contains ***one*** *compulsory question with* ***10*** *parts, having* ***2*** *parts from each unit, short answer in* ***20*** *words for each part.* ***All*** *questions carry equal marks.*

इस खण्ड में एक अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से ***2*** *लघु प्रश्न लेते हुये कुल* ***10*** *लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर* ***20*** *शब्दों से अधिक न हो।* ***सभी*** *प्रश्नों के अंक समान हैं।*

Section-B **खण्ड 'ब'** ***Max. Marks-50***

This section contains ***10*** *questions having* ***2*** *questions from each unit. Answer* ***5*** *questions (****250*** *words each) selecting* ***one*** *question from each unit.* ***All*** *questions carry equal marks.*

इस खण्ड में प्रत्येक इकाई से ***2*** *प्रश्न लेते हुये कुल* ***10*** *प्रश्न हैं। प्रत्येक इकाई से एक प्रश्न का चयन करते हुये कुल* ***5*** *प्रश्नों के उत्तर देने हैं। प्रत्येक प्रश्न का उत्तर* ***250*** *शब्दों से अधिक न हो।* ***सभी*** *प्रश्नों के अंक समान हैं।*

Section-C **खण्ड 'स'** ***Max. Marks-40***

This section contains ***4*** *descriptive type questions (questions may have sub-divisions) covering all units but not more than* ***one*** *question from each unit. Answer any* ***two*** *questions (****500*** *words each).* ***All*** *questions carry equal marks.*

इस खण्ड में ***4*** *प्रश्न वर्णनात्मक होंगे। प्रश्नों के उपभाग भी हो सकते हैं जो सभी इकाइयों में से दिये जायेंगे, किन्तु एक इकाई से एक से अधिक प्रश्न नहीं होगा। किन्हीं* ***दो*** *प्रश्नों के उत्तर दिये जाने हैं। प्रत्येक प्रश्न का उत्तर* ***500*** *शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान हैं।*

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Section A

1. (*i*) State Jenson's inequality.

(*ii*) Write slope for regression line y on x.

(*iii*) Write concept and expression for Geometric Function.

(*iv*) Write distribution function for uniform distribution.

(*v*) What is distribution of order statistics ?

(*vi*) What is maximum likelihood parameter for Poisson distribution.

(*vii*) What do you mean by consistent estimator ?

(*viii*) Let X_1, X_2, X_3 be three variate then the correlation coefficient between X_1 and X_2 after eliminating linear effect of X_3 is given by .....

(*ix*) Let (x_i, y_i) be paired sample observations $(i = 1, 2, \ldots\ldots n)$ then write hypothesis for sign test.

(*x*) What is median test ?

Section B

UNIT-I

2. State and prove central limit theorem for identically independent distribution.

Or

Two discrete random variables X and Y have joint probability density function :

$$p(x, y) = \frac{\lambda^x . e^{-\lambda} p^y (1-p)^{y-x}}{\lfloor y . \lfloor x - y} \quad \begin{matrix} y = 0,1,2,3..... \\ x = 0,1,2.3,..... \end{matrix}$$

Where λ, p are constant with $\lambda > 0$, $0 < p < 1$ Find conditional distribution of y for given x.

UNIT-II

3. If X is a Poisson variate and Y is another discrete random variable whose conditional distribution for given X is given by :

$$p(Y = r \mid X = x) = \binom{x}{r} . p^r . (1-p)^{x-r},$$

$r = 0, 1, 2, 3, \ldots\ldots x.$

then show that unconditional distribution of y $p(y)$, is a Poisson distribution with parameter mp. Where m is parameter of X.

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Or

If x and y are independent Gamma variate with parameter μ and ν respectively, show that the variate $u = x + y$, $z = \frac{x}{x+y}$ are independent and that u is $\gamma(\mu+\nu)$ and z is a $\beta(\mu, \nu)$ variates.

UNIT-III

4. Derive chi square (χ^2) distribution using moment generating function.

Or

Define method of moments. Estimate α and β in case of Pearson's type III distribution by method of moments :

$$f(x, \alpha, \beta) = \frac{\beta^n}{\sqrt{(\alpha)}} x^{\alpha-1} \cdot e^{-\beta x} \quad 0 \le x < \infty$$

UNIT-IV

5. State and prove Cramer Rao inequality.

Or

In a trivariate distribution :

$\sigma_1 = 2$, $\sigma_2 = \sigma_3 = 3$, $r_{12} = .7$, $r_{21} = r_{31} = .5$

Prove that $\sigma_{1.23} = 1.3856$, $b_{12.3} = .4$, $b_{13.2} = .1333$.

UNIT-V

6. Before an increase in excise duty on tea, 800 persons out of 1000 were tea drinker. After an increase in duty 800 people were tea drinker out of 1200 persons. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after increase in excise duty, given Z_0 (Calculated) = 1.645 at 5% level and 2.33 at 1% level of significance.

Or

Write short notes on non-parametric test :

(*a*) rank correlation test

(*b*) test of independence.

Section C

7. Two random variables have the following joint probability density function :

$$f(x, y) = \begin{cases} 2-x-y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{Otherwise} \end{cases}$$

Prove :

Marginal probability density functions of X and Y are :

(*a*) $f(x) = \frac{3}{2} - x \quad 0 < x < 1$ and 0 otherwise

$$f_y(y) = \frac{3}{2} - y \quad 0 < x < 1 \text{ and } 0 \text{ otherwise}$$

(*b*) Covariance between X and Y is $-\frac{1}{144}$.

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8. If x is a Poisson variate with mean m, show that $\frac{x-m}{\sqrt{m}}$ approaches to normal variate as $x \to \infty$.

9. Variates x and y have the joint p.d.f. given by $f(x, y) = \frac{1}{3}(x+y)$ $0 \leq x \leq 1$, $0 \leq y \leq 2$. Find :

(*i*) Correlation coefficient

(*ii*) Two lines of Regressions.

10. If the normal variates x_1 and x_2 are independent and follows χ^2 distributions with n *d.f.* Show that $\sqrt{n}\ (x_1 - x_2)/\sqrt{x_1.x_2}$ is distributed as student's t variate with n *d.f.*

11. Define most efficient estimator.

A random sample $(x_1, x_2, x_3, x_4, x_5)$ of size 5 is drawn from a normal population with unknown mean consider the following estimators to estimate μ :

(*i*) $\frac{1}{5}\ (x_1 + x_2 + x_3 * x_4 + x_5) = t_1$

(*ii*) $\frac{x_1 + x_2}{2} + x_3 = t_2$

(*iii*) $\frac{2x_1 + x_2 + \lambda x_3}{3} = t_3$, where λ is such that t_3 is unbiased. Find λ, are t_1 and t_2 independent, Out of t_1, t_2, t_3 which one is best estimator of μ.

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