1656

M.A./M.Sc. (Final) Examination, 2020

MATHEMATICS

Paper VI (Comp.)

(Functional Analysis)

Time Allowed : Three Hours

Maximum Marks : 100

*This question paper contains **three** sections as under :*

Section-A **खण्ड 'अ'** **Max. Marks-10**

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*

*इस खण्ड में **एक** अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से **2** लघु प्रश्न लेते हुए कुल **10** लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर **20** शब्दों से अधिक न हो। **सभी** प्रश्नों के अंक समान हैं।*

Section-B **खण्ड 'ब'** **Max. Marks-50**

*This section contains **10** questions having 2 questions from each unit. Answer **5** questions (250 words each) selecting **one** question from each unit. **All** questions carry equal marks.*

*इस खण्ड में प्रत्येक इकाई से **2** प्रश्न लेते हुए कुल **10** प्रश्न हैं। प्रत्येक इकाई से **एक** प्रश्न का चयन करते हुए, कुल **5** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न का उत्तर **250** शब्दों से अधिक न हो। **सभी** प्रश्नों के अंक समान हैं।*

Section-C **खण्ड 'स'** **Max. Marks-40**

*This section contains **4** descriptive type questions (questions may have sub-divisions) covering all units but not more than one question from each unit. Answer any **two** questions (**500** words each). **All** questions carry equal marks.*

*इस खण्ड में **4** वर्णनात्मक प्रश्न होंगे (प्रश्नों के उप-भाग भी हो सकते हैं) जो सभी इकाइयों में से दिये जायेंगे, किन्तु एक इकाई से एक से अधिक प्रश्न नहीं होगा। किन्हीं दो प्रश्नों के उत्तर दिये जाने हैं। प्रत्येक प्रश्न का उत्तर **500** शब्दों से अधिक न हो। **सभी** प्रश्नों के अंक समान हैं।*

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Section A

1. (*i*) Define diameter of a set.

(*ii*) Define closed ball.

(*iii*) Define convergent sequence in metric space.

(*iv*) Define fixed point.

(*v*) Define Banach space.

(*vi*) Define norm of a continuous linear transformation.

(*vii*) Define inner product space.

(*viii*) State Cauchy Schwarz's inequality.

(*ix*) Define orthogonal complement.

(*x*) Define self-adjoint operator.

Section B

UNIT-I

2. Let C be the set of all real valued bounded continuous functions defined on [0, 1]. If d is defined by $d(f, g) = \sup\{|f(x) - g(x)| : x \in [0, 1]\}$, $f, g \in C$, then show that (C, d) is metric space.

3. In a metric space, show that each open sphere is an open set.

UNIT-II

4. Show that every complete metric space is of second category as a subset of itself.

5. Show that every contracting mapping is continuous mapping.

UNIT-III

6. State and prove Minkowski's inequality.

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7. Let N and N′ be normed linear space, then show that the set B(N, N′) of all continuous linear transformations from N into N′ is itself a normed linear space with respect to pointwise linear operations $(T_1 + T_2)x = T_1(x) + T_2(x)$, $(\alpha T)(x) = \alpha T(x)$ and the norm is defined by $\|T\| = \sup\{\|T(x)\| : x \in N, \|x\| \leq 1\}$.

UNIT-IV

8. State and prove open mapping theorem.

9. Prove that every inner product space is a normed linear space.

UNIT-V

10. Let M and N be closed linear subspace of a Hilbert space H, such that $M \perp N$. Then show that the linear subspace M + N is also closed.

11. If T is an operator on a Hilbert space H, then show that $(Tx, x) = 0 \ \forall \ x \in H \Leftrightarrow T = 0$.

Section C

12. (*a*) Let A be a subset of a metric space (X, *d*). Then show that a point $x \in X$ is an adherent point of A if and only if $d(x, A) = 0$.

(*b*) If a Cauchy sequence in a metric space has a convergent sub-sequence, then show that the sequence is convergent.

13. (*a*) Let Y be a sub-space of a complete metric space (X, *d*). Then show that Y is complete $\Leftrightarrow$ Y is closed.

(*b*) Let N and N′ be normed linear space with same scalars and let $T : N \to N'$ be a linear transformation, then show that T is bounded $\Leftrightarrow$ T is continuous.

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14. (*a*) Show that a non-empty subset X of a normed linear space N is bounded if and only if $f(X)$ is a bounded set of numbers for each f in N^* (conjugate space of N).

(*b*) Show that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

15. (*a*) State and prove Bessel's inequality.

(*b*) If M and N are closed linear sub-spaces of a Hilbert space H, P and Q are projections on M and N respectively, then show that :

$$M \perp N \Leftrightarrow PQ = 0 \Leftrightarrow QP = 0.$$

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