M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper VI (Comp.)

(Functional Analysis)

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This section contains ***one*** *compulsory question with 10 parts, having 2 parts from each unit, short answer in 20 words for each part. All questions carry equal marks.*

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Section-B ***Max. Marks-50***

This section contains 10 questions, having 2 questions from each unit. Answer ***five*** *questions (250 words each) selecting one question from each unit. All questions carry equal marks.*

Section-C ***Max. Marks-40***

This section contains 4 descriptive type questions (questions may have sub-divisions) covering all units but not more than ***one*** *question from each unit. Answer any* ***two*** *questions (500 words each). All questions carry equal marks.*

Section A

1. (i) Define bounded set.

(ii) Define interior point of a set.

(iii) Define complete metric space.

(iv) Define fixed point.

(v) Define Banach space.

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(vii) Define continuous linear transformation

(viii) State closed graph theorem

(viii) State parallelogram law

(ix) State Bessel's inequality

(x) Define Operator

Section B

UNIT-I

2. State and prove Minkowski's inequality

3. Show that the set C of all complex numbers is a metric space under the metric defined by

$$d(z_1, z_2) = \frac{|z_1 - z_2|}{(1 + |z_1|^2)^{1/2}(1 + |z_2|^2)^{1/2}}$$

UNIT-II

4. State and prove Cantor's intersection theorem.

5. Let (X, d) be a complete metric space and let f be a contracting mapping on X, then show that there exists unique x in X such that $f(x) = x$.

UNIT-III

6. Show that every normed linear space is a metric space.

7. Show that the linear space C(x) of all bounded continuous scalar valued function f defined on X as a Banach space under the norm defined by $\|f\| = \sup \{|f(x)| : x \in X\}$.

UNIT-IV

8. State and prove open mapping theorem.

9. Show that every inner product space is a normed linear space.

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UNIT-V

10. Let S be a non-empty subset of a Hilbert space H, then show that $S^{\perp}$ is a closed linear subspace of H.

11. Show that an operator T on a Hilbert space H is self adjoint if and only if (Tx, x) is real for all x.

Section C

12. (*a*) Show that in a metric space each open sphere is an open set.

(*b*) Let A be a subset of a metric space, then show that :

$$\overline{A} = A \cup D(A).$$

13. (*a*) Show that every convergent sequence in a metric space is a Cauchy sequence but converse need not be true.

(*b*) Show that a linear transformation T is continuous if and only if T is bounded.

14. (*a*) Show that a non-empty subset X of a normed linear space N is bounded if and only if $f(X)$ is bounded set of numbers for each f in N^* (conjugate space of N).

(*b*) Let M be a closed linear subspace of a normed linear space N and let x_0 be a vector not in M. If d is the distance of x_0 to M, then show that there exist f_0 in N^* such that :

$$f_0(M) = \{0\},\ f_0(x_0) = d \text{ and } \|f_0\| = 1.$$

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15. (*a*) Let T be an arbitrary operator on a Hilbert space H. Then show that T can be uniquely expressed as $T = T_1 + iT_2$, where T_1 and T_2 are self adjoint operators in H.

(*b*) Let P be a projection on a Hilbert space H with range M and null space N, then show that :

$$M \perp N \Leftrightarrow P \text{ is self adjoint.}$$

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