U-1656

M. A./M.Sc. (Final) Examination, 2021

MATHEMATICS

Paper - VI

FUNCTIONAL ANALYSIS

Time Allowed : **Three Hours**

Maximum Marks : **100**

Note : *This question paper contains* ***three*** *sections as under :*

Max. Marks-10

:tion-A

This section contains **one compulsory** question :ith 10 parts, having 2 parts from each unit, short answer in 20 words for each part. All questions carry equal marks.

Section-B **Max. Marks-50**

This section contains **10** questions having 2 questions from each unit. Answer 5 questions (250 words each) selecting one question from each unit. All questions carry equal marks.

Section-C **Max. Marks-40**

This section contains 04 descriptive type questions (questions may have sub-divisions) covering all units but not more than one question from each unit. Answer any two questions. (500 words each). All questions carry equal marks.

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SECTION - A

1 (i) Give an example of a pseudo metric which is not a metric.

(ii) Define the accumulation point of a metric space.

(iii) Define the complete Metric spaces.

(iv) Define the contracting mapping.

(v) Define normed linear space.

(vi) Define the jointly continuous function.

(vii) Define Hilbert space.

(viii) State Cauchy Schwarz Inequality.

(ix) State projection theorem.

(x) Define orthonormal set.

SECTION - B

UNIT - I

2 Define a closed set. In a metric space every closed sphere is a closed set.

3 Every convergent sequence in a metric space is a Cauchy sequence but convergs need not be true.

UNIT - II

4 Let (Y, d) be a subspace of a complete metric space (X, d), then Y is complete iff Y is closed.

5 Every complete metric space is of the second category as a subset of itself.

UNIT - III

6 Let N and N' be normed linear spaces and $T : N \to N'$ be a linear transformation. Then T^{-1} exists and is continuous on its domain iff there exists $K > 0$ such that

$$K\|x\| \leq T\|x\|, \forall x \in N$$

7 Define Banach space. Let B and B' be Banach spaces and T be a one-one continuous linear transformation of B onto B'. Then T is a homomorphism. In particular T^{-1} is automatically continuous.

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UNIT - IV

8 Let B and B' the two Banach space and T be a linear transformation of B into B'. Then T is continuous iff its graph is closed.

9 Define inner product space. If x and y are any two vectors in a Hilbert space H, then $|(x,y)| \leq \|x\| \|y\|$.

UNIT - V

10 Define orthogonal complement. Let M be a linear subspace of a Hilbert space H, then M is closed $M = M^{\perp\perp}$.

11 Let $\{e_i\}$ be an orthonormal set in a Hilbert space H and x is any vector in H, then $\sum |(x,e_i)|^2 \leq \|x\|^2$.

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SECTION - C

12 (a) State and prove open mapping theorem.

(b) Let $\{x_1, x_2, \ldots\ldots x_n, \ldots\ldots\}$ be a linear independent set of vectors in a Hilbert space H. Then there exists an orthonormal set of vectors $\{e_1, e_2, \ldots\ldots e_n, \ldots\ldots\}$ such that, for any $n\left[\{x_1, x_2, \ldots\ldots, x_n\}\right] = \left[\{e_1, \ldots\ldots, e_n\}\right]$ i.e. for any n, the linear subspace of H spanned by the vectors $x_1, x_2, \ldots\ldots x_n$ is the same as that spanned by the vectors $e_1, e_2, \ldots\ldots e_n$. https://www.uokonline.com

13 (a) Let M be a closed linear subspace of a normed linear space N. If the norm of a coset $x + M$ in the quotient space N/M is defined by $\|x+M\| = \inf\{\|x+m\| : m \in M\}$ then N/M is a normed linear space. Further if N is a Banach space, then N/M is also a Banach space.

(b) State and prove Banach fixed point theorem.

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14 (a) Let N and N' be normed linear spaces, then the set of $B(N, N')$ of all continuous (or bounded) linear transformations from N into N' is itself a normed linear space with respect to pointwise linear operations

$$(T_1+T_2)x=T_1(x)+T_2(x), \qquad (\alpha T)x=\alpha T(x)$$

and the norm is defined by

$$\|T\|=\text{Sup}\{\|T(x)\|:x\in N, \|x\|\leq 1\}$$

Further if N' is a Banach space, then $B(N, N')$ is also a Banach space.

(b) The self adjoint operators in $B(H)$ form a closed linear subspace and therefore a real Banach space which contains the identity transformation.

15 (a) Let P is a projection on a closed linear subspace M of H iff $I-P$ is the projection on $M^{\perp}$.

(b) State and prove Banach–Steinhous theorem.

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