This question paper contains 8 printed pages]

**1658**

# M.A./M.Sc. (Final) Examination, 2020

## MATHEMATICS

## Paper I

## (Optional)

## (Operations Research)

**Time Allowed : Three Hours**

**Maximum Marks : 100**

***Note** : Non-Programmable Scientific Calculator is allowed in this paper.*

*This question paper contains **three** sections as under :*

***Section-A*** ***Max. Marks-10***

*This section contains **one** compulsory question with **10** parts, having **2** parts from each unit, short answer in **20** words for each part. **All** questions carry equal marks.*



***Section-B*** ***Max. Marks-50***

*This section contains **10** questions having **2** questions from each unit. Answer **5** questions (**250** words each) selecting **one** question from each unit. **All** questions carry equal marks.*

***Section-C*** ***Max. Marks-40***

*This section contains **4** descriptive type questions (questions may have sub-divisions) covering all units but not more than one question from each unit. Answer any two questions (**500** words each). **All** questions carry equal marks.*

### Section A

1. (*i*) Write any *two* advantages of dual simplex method over simplex method.

(*ii*) Write dual of :

$$\text{Max. : } Z = CX$$

$$AX \leq b$$

$$X \geq 0$$

(*iii*) Define two person zero sum game.



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(*iv*) State minimax-maximin principle for mixed strategy game.

(*v*) Define free float.

(*vi*) State Bellman's principle of optimality.

(*vii*) Write objective of inventory cost control.

(*viii*) Write Kuhn-Tucker necessary conditions for the problem :

$$\text{Max. : } f(X)$$

Subject to :

$$g_i(X) \geq 0 \quad i = 1, 2, \ldots\ldots m$$

$$g_i(X) \leq 0 \quad i = m + 1, m + 2, \ldots\ldots p$$

$$h_j(X) = 0 \quad j = 1, 2, \ldots\ldots q$$

$$X \geq 0$$

(*ix*) What do you mean by steady state in queuing models ?

(*x*) Define traffic intensity.



## UNIT-I

2. Solve the following problem by Two-Phase method :

$$\text{Min. : } Z = 6x_1 + 4x_2$$

$$3x_1 + 3x_2 \geq 40$$

$$2x_1 + 5x_2 \geq 44$$

$$3x_1 + x_2 \geq 40$$

$$x_1, x_2 \geq 0$$

3. Use Duality to solve :

$$\text{Max. : } Z = 2x_1 + 3x_2 + 5x_3$$

$$x_1 + x_2 + x_3 \leq 7$$

$$x_1 + x_2 + 2x_3 \leq 13$$

$$3x_1 - x_2 + x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0$$

## UNIT-II

4. Solve the game :

| | I | II | III | IV |
|---|---|---|---|---|
| I | 3 | 2 | 4 | 0 |
| II | 3 | 4 | 2 | 4 |
| III | 4 | 2 | 4 | 0 |
| IV | 0 | 4 | 0 | 8 |



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5. Solve the Integer programming problem by branch and bound technique :

$$\text{Max. : } Z = x_1 + x_2$$

$$3x_1 + 2x_2 \leq 12$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0 \quad \text{and are integers}$$

## UNIT-III

6. Find the critical path and project completion time, given :

| Jobs | Predecessor | Time (in Hours) |
|---|---|---|
| A | — | 2 |
| B | C | 5 |
| C | — | 1 |
| D | A, B | 3 |
| E | C | 1 |
| F | D | 4 |
| G | E, F | 3 |
| H | D | 2 |

7. Divide a positive quantity '$b$' into $n$ parts so as to maximize their product.



## UNIT-IV

8. The demand for an item is 8000 units per annum and the unit cost is ₹ 1, inventory carrying charges of 20% of average inventory cost and ordering cost is ₹ 12.50 per order. Calculate optimal order quantity, optimal order time, optimal inventory cost and number of orders.

9. Solve :

$$\text{Optimize : } Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

$$x_1 + x_2 + x_3 = 15$$

$$2x_1 - x_2 + 2x_3 = 20$$

$$x_1, x_2, x_3 \geq 0.$$

## UNIT-V

10. In a transport company, trucks of the company arrive at a transhipment yard for unloading in a pattern which is characterized by the Poisson distribution. The average rate of arrival is 5 per hour. A batch of loadmen unload the parcels from the trucks and level of service is 8 trucks per hour an average. The driver is paid ₹ 100



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per shift of 8 hours and the batch of loadmen paid ₹ 72 per shift of 8 hours. Calculate :

(*i*) The cost of Idle time for driver.

(*ii*) The total cost of one, two and three batches of loadman.

(*iii*) How many batches of loadman is sufficient ?

11. Derive expressions for pure birth process.

## Section C

12. Solve the following problem by simplex method :

$$\text{Max. : } Z = 10x_1 + 3x_2 + 6x_3 + 5x_4$$

$$x_1 + 2x_2 + x_4 \leq 6$$

$$3x_1 + 2x_3 \leq 5$$

$$x_2 + 4x_3 + 5x_4 \leq 3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Also compute the limit of $q_{13}$ and $a_{11}$ so that optimal solution of above problem remains an optimal feasible solution.



13. Find optimal integer solution of integer programming problem :

$$\text{Max. : } Z = 3x_1 + 4x_2$$

$$3x_1 + 2x_2 \leq 8$$

$$x_1 + 4x_2 \leq 10$$

$$x_1, x_2 \geq 0 \text{ and are integers}$$

14. A small project is composed of 7 activities. Time estimates are listed below :

(*i*) Draw the network.

(*ii*) Calculate the expected variance.

(*iii*) Calculate total float, free float, independent float.

(*iv*) Calculate the probability that project will be completed at least 3 weeks earlier than expected.

(*v*) If project due date is 18 weeks, what is the probability of not meeting due date ?

15. Using Wolfe's method solve :

$$\text{Max. : } Z = 2x_1 + x_2 - x_1^2$$

$$\text{S.t. : } 2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$



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