QZ-1656

M. A. / M. Sc. (Final)
Examination, 2024

MATHEMATICS

Paper - VI

FUNCTIONAL ANALYSIS

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains three sections as under :

Section-A खण्ड–अ **Max. Marks-10**

This section contains one compulsory question with 10 parts, having 2 parts from each unit, short answer in 20 words for each part. All questions carry equal marks.

इस खण्ड में एक अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से 02 लघु प्रश्न लेते हुये कुल 10 लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर 20 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान है।

Section-B खण्ड–ब **Max. Marks-50**

This section contains 10 questions having 2 questions from each unit. Ansuer 5 questions (250 words each) selecting one question from each unit. All questions carry equal marks.

इस खण्ड में प्रत्येक इकाई से 02 प्रश्न लेते हुये कुल 10 प्रश्न है। प्रत्येक इकाई से एक प्रश्न का चयन करते हुये कुल 05 प्रश्नो के उत्तर देने है। प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान है।

Section-C खण्ड–स **Max. Marks-40**

This section contain 4 descriptive type questions (questions may have sub division) covering all units but not more than one question from each unit. Answer any two questions. (500 words each). All questions carry equal marks.

इस खण्ड में 4 वर्णनात्मक प्रश्न होंगे। प्रश्न में भाग भी हो सकते हैं जो सभी इकाईयों में से दिये जायेगें, किन्तु एक इकाई में से एक से अधिक प्रश्न नहीं होगा। कोई दो प्रश्नों के उत्तर दिये जाने है। प्रत्येक प्रश्न का उत्तर 500 शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान हैं।

https://pyqbook.com

SECTION - A

1 (i) Define Metric Space.

(ii) Define limit point of a set.

(iii) State Baine category theorem.

(iv) What do you mean by contracting mapping?

(v) Define Normed Linear Space.

(vi) Define Continuous linear transformation.

(vii) State Open mapping theorem.

(viii) State Cauchy Schwartz's inequality.

(ix) Define Orthonormal sets.

(x) Define Self adjoint operator.

SECTION - B

UNIT - I

2 If X is a set of all continuous real valued function defined on interval [0, 1] and $d : X \times X \to R$ is defined as follow

$$d(f, g) = \text{maximum}\{|f(x) - g(x)| : x \in [0, 1]\}$$

$\forall f, g \in X$ then show that (X, d) is a metric space.

3 Prove that every sphere in any metric space is an open set.

UNIT - II

4 If X and Y are any two metric spaces. Prove that a mapping $f : X \to Y$ is continuous on X if and only if for any open subset G of Y, $f^{-1}(G)$ is open in X.

5 Prove that any sub-metric space (Y, d) of a metric space (X, d) is complete if and only if Y is closed.

UNIT - III

6 If $x = (x_1\ x_2 \ldots\ldots x_n)$ and $y = (y_1\ y_2 \ldots\ldots y_n)$ be two n-tuples of real or complex numbers then for norm

$$\|x\|_p = \left[\sum_{i=1}^{n} |x_i|^p\right]^{1/p}, p \geq 1$$

Prove that $\|x + y\|_p \leq \|x\|_p + \|y\|_p$

https://pyqlelp.com

7 If N be a Normed linear space and x_0 is a non-zero vector then show that there exists a continuous linear functional F defined on conjugate space N* such that $F(x_0) = \|x_0\|$ and $\|F\| = 1$.

UNIT - IV

8 If B be a Banach space and N be a Normed linear space, $\{T_G\}$ a non-empty set of bounded and continuous linear transformations of B into N, prove that $\{T_i(x)\}$ is a bounded subset of N for each vector x in B, the $\{\|T_i\|\}$ is bounded set of numbers.

9 State and prove parallelogram law for Hilbert space.

UNIT - V

10 If M be a proper closed linear subspace of a Hilbert space H then prove that there exists a non-zero vector Z_0 in H such that $Z_0 \perp M$.

11 If T is an operator on Hilbert space H then prove that $(T_x, x) = 0 \ \forall x \in H$ if and only if $T = 0$.

SECTION - C

12 State and prove Fixed point theorem.

13 Define Banach space. If p is a real number $(1 \le p < \infty)$ show that the space l_p^n of all n-tuples of scalers with norm defined by $\|x_p\| = \left\{\sum_{i=1}^{n} |x_i|^p\right\}^{1/p}$ is a Banach space.

https://pyqlelo.com

14 If B and B' are Banach space and T is a linear transformation of B into B' then prove that T is continuous if and only if its graph is closed.

15 If $\{e_i\}$ be an orthonormal set in a Hilbert space H and x be an arbitrary vector in H then prove that

$x - \sum (x, e_i) e_i \perp e_i$ for all i.

---

https://pyqlelo.com