KAF-1656

M. A. / M. Sc. (Final)
Examination, 2025

MATHEMATICS

Paper - VI

FUNCTIONAL ANALYSIS
(Only for Due./Ex. Students)

Time Allowed : **Three Hours**

Maximum Marks : **100**

This question paper contains ***three*** *sections as under :*

SECTION - A

1 (i) Define Pseudo Metric.

(ii) Define interior point of a set.

(iii) Define nowhere dense set.

(iv) State Banach fixed point theorem.

(v) Define Banach space.

(vi) State Hahn - Banach theorem.

(vii) Define orthogonal complement of a set.

(viii) Define projection.

(ix) Define unitary space.

(x) Write standard inner product for R^n.

SECTION - B

UNIT - I

2 Prove that any sequence in a metric space converges to a unique point.

https://pyqiele.com

3 Let $\langle x, d \rangle$ be a metric space and Let d^* be defined as

$$d^*(x, y) = \frac{M\, d(x, y)}{1 + d(x, y)}, \quad \forall\, x, y \in X,\ M > 0$$

Then show that $\langle x, d^* \rangle$ is also a metric space.

UNIT - II

4 Show that every complete metric space is of the second category as a subset of itself.

5 Show that contracting mapping is a continuous mapping.

UNIT - III

6 In a normed linear space, prove the following:

(a) $\big|\, \|x\| - \|y\| \,\big| \le \|x - y\|$

(b) Norm is a continuous function

i.e. $x_n \to x \Rightarrow \|x_n\| \to \|x\|$

7 Let N and N' be normed linear spaces and $T : N \to N'$ be a linear transformation. Then T^{-1} exists and is continuous on its domain iff there exists $k > 0$ such that

$k \|x\| \le \|T(x)\|,\ \forall\, x \in N$.

UNIT - IV

8 State and prove Cauchy-Schwarz's inequality for Hilbert space.

9 State and prove open mapping theorem.

UNIT - V

10 Let S, S_1, S_2 be non empty subset of a Hilbert space H, then

(a) $H^{\perp} = \{0\}$

(b) $\{0\}^{\perp} = H$

(c) $S \cap S^{\perp} = \{0\}$

(d) $S_1 \subset S_2 \Rightarrow S_2^{\perp} \subset S_1^{\perp}$

https://www.uokonline.com

11 If T is an operator on a Hilbert space H, then $\langle Tx, x\rangle = 0$ for each $x \in H$ iff $T = 0$ (zero operator).

SECTION - C

12 State and prove cantor intersection theorem.

13 Show that the linear space $C[0,1]$ of real valued continuous functions f on [0, 1] is a normed linear space but not a Banach space under the norm defined by -

$$\|f\| = \int_0^1 |f(t)|\, dt$$

14 State and prove the uniform boundedness theorem.

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15 Let $\{e_1, e_2, \ldots\ldots, e_n\}$ be a finite orthonormal set in a Hilbert space H. If x is any vector in H, then

$$\sum_{i=1}^{n} |\langle x, e_i\rangle|^2 \le \|x\|^2$$

Further $x - \sum_{i=1}^{n} \langle x, e_i\rangle e_i \perp e_j, \forall j$

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